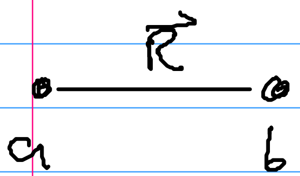
Van der Waals interaction
$\longrightarrow$ Casimir potentials

Interactions between neutral objects



$$H_{int} = \iint \frac{\rho_a \, \rho_b}{r_{ab}} \, d^3 r_a \, d^3 r_b$$

Multipole expansion

$$H' = \frac{1}{R^3}\left(\vec{d}_a \cdot \vec{d}_b - 3\left(\vec{d}_a \cdot \hat{R}\right)\left(\vec{d}_b \cdot \hat{R}\right)\right)$$

Classical: $\langle d \rangle = 0 \qquad \langle d^2 \rangle = 0$

It cost more energy to create a dipole moment than one gets back from the dipole interaction.

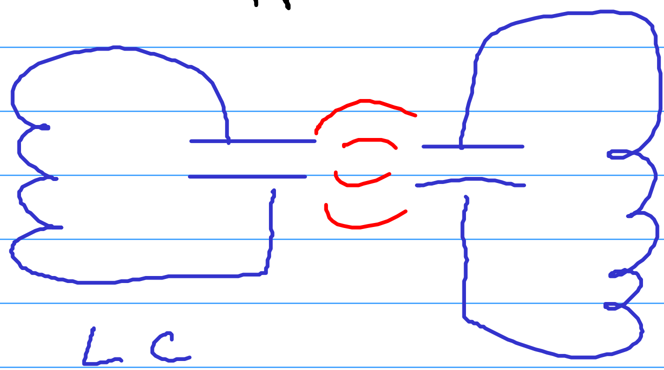
QM: $\langle d^2 \rangle \neq 0$

1st order perturbation theory $\quad -\dfrac{C_3}{R^3} \qquad g \leftrightarrow e$

2nd order $\quad -\dfrac{C_6}{R^6} \qquad g \leftrightarrow g$

Interpretation

See D. Kleppner



two coupled capacitors coupling $\propto \frac{1}{R^3}$

$$\omega_\pm = \omega_0 \pm (\ )\frac{1}{R^3} - (\ )\frac{1}{R^6}$$

QM: osc. in ground state
zero-point energy

$$W = \frac{1}{2}\hbar(\omega_+ + \omega_-) = \text{Const} - (\ )\frac{1}{R^6}$$

vdW potential is due to the zero point energy of the atomic oscillators

- equivalent description
  $\langle d_a \rangle = 0$, but $\langle d_a^2 \rangle \neq 0$

$$E_b \sim \frac{d_a}{R^3} \quad \text{fluctuating field}$$

$\Rightarrow$ induces dipole moment $d_b \propto E_b \sim \frac{d_a}{R^3}$

dipole-dipole interaction $\frac{d_b d_a}{R^3} \propto \frac{\langle d_a^2 \rangle}{R^6}$

$\frac{1}{R^6}$ potential is caused by zero-point fluctuations of the atomic dipole moment

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Now: Vacuum Fluctuations of the em field
$\Rightarrow$ correlated dipole moments

$$d_a = \alpha_a E(r_a)$$
$$d_b = \alpha_b E(r_b)$$

Dipole-dipole interaction
$$\propto \frac{\alpha_a \alpha_b \langle \vec{E}(r_a) \vec{E}(r_b) \rangle}{R^3}$$

Use density of modes and zero-point energy for the e.m field
(Spruch p 40 ff) $\longrightarrow \frac{1}{R^7}$

$\frac{1}{R^6}$	Uncertainty principle for atoms	Short
$\frac{1}{R^7}$	" " " em Waves	Long

RANGE

BUT: Both quantum descriptions necessary for consistent description

NOW: diagrammatic approach

Virtual photon exchange

$$H_I' = -d\, \underset{\sim\sim\sim}{E_\perp(R)} - d'\, \underset{\sim\sim\sim}{E_\perp(R')}$$

No direct Coulomb interaction

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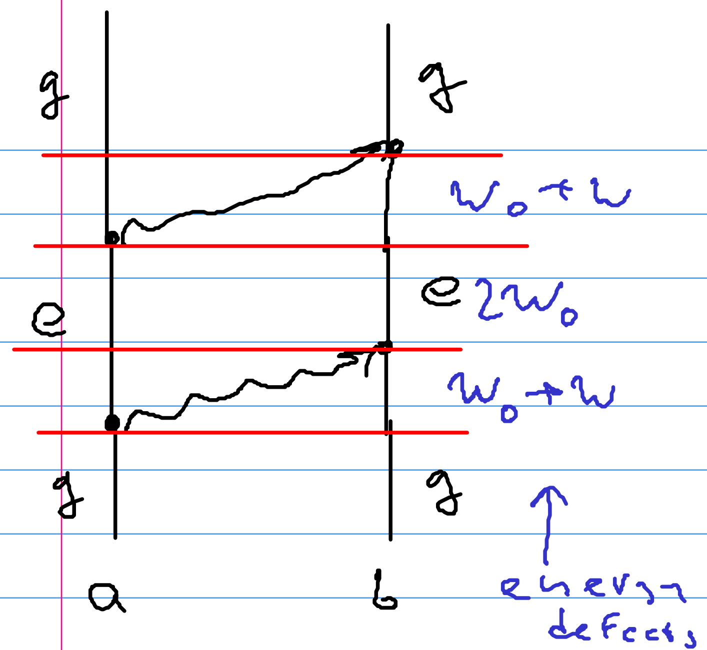
- two atoms at distance D
- dominant contribution from virtual photons with $\lambda = D$, $\omega = \frac{2\pi c}{D}$

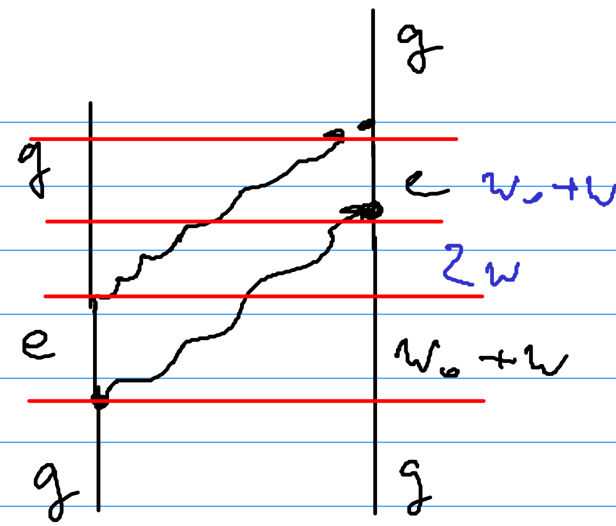
  density of states favours high ω

  but $\lambda \ll D$ cancel due to e^{ikR} terms
- higher order perturbation theory

$$\Delta E \approx \frac{V_f \ldots V_{21} V_{1i}}{\Delta E_n \ldots \Delta E_1}$$ ← energy defect

- $\hbar\omega_0 = E_e - E_g$





short $w \gg w_0$ $D < \lambda_{ge}$

$$\frac{1}{w^2 w_0} \qquad \cancel{\frac{1}{w^3}}$$

long RANGE $w_0 \gg w$

$$\cancel{\frac{1}{w_0^3}} \qquad \frac{1}{w_0^2 w}$$

Long range: additional factor of $w \propto \frac{1}{D}$

$$\frac{1}{D^6} \xrightarrow[\text{long range}]{} \frac{1}{D^7}$$

instantaneous $\rightarrow$ retarded potential

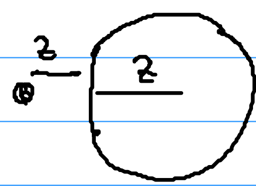
CASIMIR INTERACTION

See Spruch paper

atom-atom $\quad V \propto \dfrac{\alpha_1 \alpha_2}{R^7} \qquad \dfrac{1}{R^6}$

long range $\qquad$ short range

atom-wall

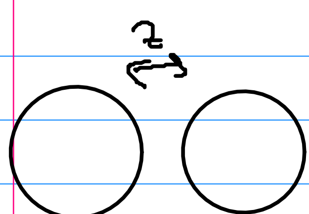


conducting sphere with radius z

$\alpha_{sphere} \propto z^3$

$$\Rightarrow V \propto \frac{\alpha_1}{z^4} \qquad \frac{1}{z^3}$$

two walls



$\alpha_1 \propto z^3$

$\alpha_2 \propto z^3$

$$\frac{V}{\text{Area}} \propto \frac{1}{z}\frac{1}{z^2} = \frac{1}{z^3}$$

no short distance

Casimir potential $\dfrac{1}{z^3}$