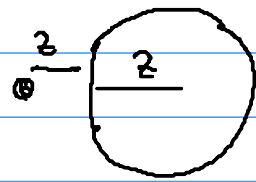
CASIMIR INTERACTION

See Spruch paper

atom-atom $\quad V \propto \dfrac{\alpha_1 \alpha_2}{R^7} \qquad \dfrac{1}{R^6}$

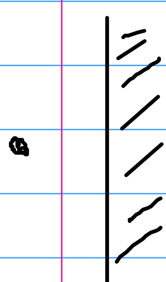
long range $\qquad$ short range



conducting sphere with radius z

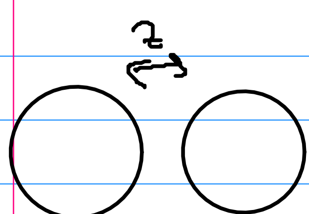
$\alpha_{sphere} \propto z^3$

atom-wall



$\Rightarrow V \propto \dfrac{\alpha_1}{z^4} \qquad \dfrac{1}{z^3}$

two walls



$\alpha_1 \propto z^3$

$\alpha_2 \propto z^3$

$\dfrac{V}{\text{Area}} \propto \dfrac{1}{z}\dfrac{1}{z^2} = \dfrac{1}{z^3}$

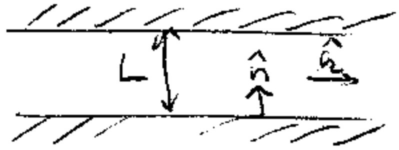
no short distance

Casimir potential $\dfrac{1}{z^3}$

CASIMIR EFFECT (Cf. S. Haroche Les Houches Summer School

Attractive force between two parallel conducting plates due to vacuum fluctuations of the e.m. field

e.m. modes in cavity width L, cross section a^2, $a \gg L$



boundary conditions at $z=0$ and $z=L$

$$\sin L\ell = 0$$
$$\ell = m\pi/L \qquad m = 0, 1, 2, \ldots$$
$$\omega^2 = k^2c^2 + m^2\pi^2c^2/L^2$$

$$\text{TE modes: } \vec{A}^{E}_{\vec{k},m} = \sin\left(\frac{m\pi z}{L}\right)\left(\hat{k}\times\hat{n}\right)e^{ik\hat{k}\vec{\rho}} + \text{c.c.} \qquad m \geq 1$$

$$\text{TM modes } \vec{A}^{M}_{\vec{k},m} = \left\{\left[\frac{ck}{\omega}\cos\left(\frac{m\pi z}{L}\right)\hat{n} - \frac{im\pi c}{L\omega}\sin\left(\frac{m\pi z}{L}\right)\hat{k}\right] \times e^{ik\hat{k}\vec{\rho}}\right\} + \text{c.c.} \qquad m \geq 0$$

o Density of states

- number of modes with given m and k between k and $k+dk$

$$\frac{dk_x}{\left(\frac{2\pi}{a}\right)}\frac{dk_y}{\left(\frac{2\pi}{a}\right)} = \frac{a^2\,k\,dk}{2\pi} = \frac{a^2}{4\pi}\,d(k^2) = \frac{a^2}{c^2}\frac{\omega\,d\omega}{2\pi}$$

- for a given ω: $m = 0, \ldots, \operatorname{Int}\left(\frac{\omega L}{\pi c}\right)$

$$\rho(\omega)\,d\omega = \frac{a^2}{2\pi c^2}\left[1 + 2\operatorname{Int}\left(\frac{\omega L}{\pi c}\right)\right]\omega\,d\omega$$

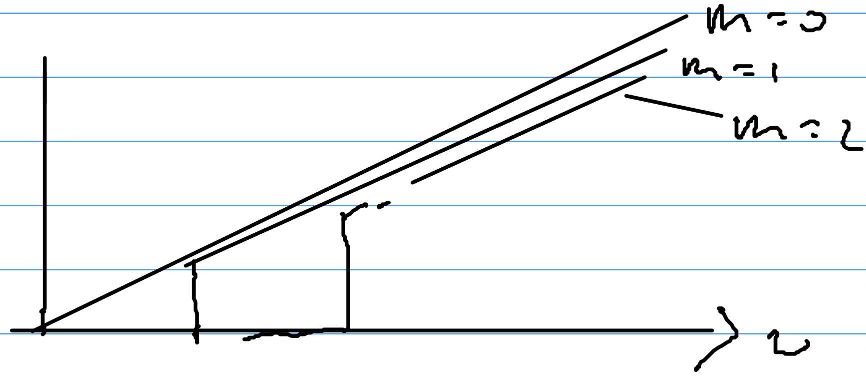
$$= \frac{a^2}{2\pi c^2}\left[1 + 2\sum_{m=1}^{\infty}\Theta\left(\omega - m\frac{\pi c}{L}\right)\right]\omega\,d\omega$$

Heaviside step function

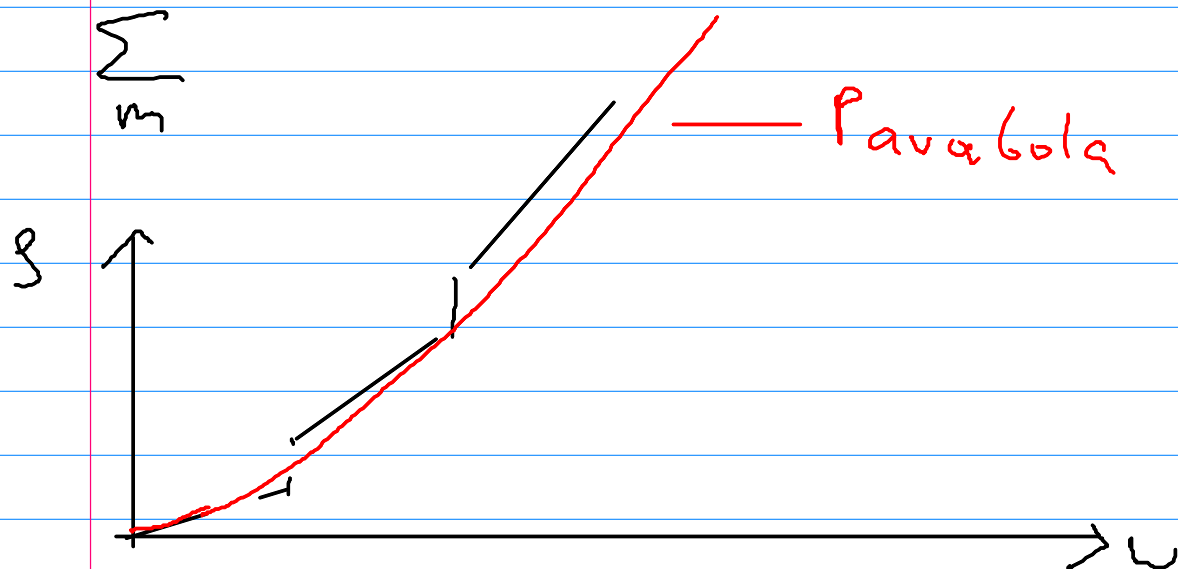
Density of modes for a given m

$(\,) \, \omega \, d\omega$ $\qquad \uparrow$

$\overline{\underline{g}}$



$\sum_m$



Zero-point energy in the volume $a^2 \times L$

$$W(L) = \int_0^\infty \frac{\hbar\omega}{2}\, \rho(\omega)\, d\omega$$

$$= \frac{a^2\hbar}{4\pi c^2}\left[\int_0^\infty \omega^2 d\omega + 2\sum_{m=1}^{\infty}\int_{\frac{m\pi c}{L}}^{\infty} \omega^2 d\omega\right]$$

add convergence term $e^{-\lambda\omega/c}$

[cutoff at high frequencies $\omega \gg \lambda/c$]

$$W(L) = \text{const.} + \frac{a^2\hbar}{2\pi c^2}\sum_m I_m$$

with $I_m = \int_{\frac{m\pi c}{L}}^{\infty} \omega^2 e^{-\lambda\omega/c}\, d\omega$

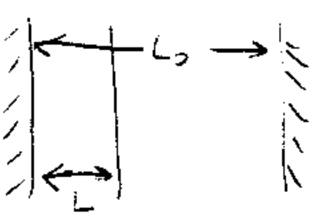
$$I_m = c^2 \frac{\partial^2}{\partial\lambda^2}\int_{\frac{m\pi c}{L}}^{\infty} e^{-\lambda\omega/c}\, d\omega = c^3 \frac{\partial^2}{\partial\lambda^2}\left[-\frac{e^{-m\pi\lambda/L}}{\lambda}\right]$$

$$\sum_m I_m = \frac{c^3\pi}{L}\frac{\partial^2}{\partial\lambda^2}\left[\frac{1}{\left(\frac{\pi\lambda}{L}\right)}\,\frac{1}{e^{(\pi\lambda/L)}-1}\right]$$

Expansion around $x = 0$ ($x = \pi\lambda/L$, eventually $\lambda \to 0$)

$$\frac{1}{x(e^x-1)} = \frac{1}{x^2} - \frac{1}{2x} + \frac{1}{12} - \frac{x^2}{720} + \ldots$$

$$W(L) = W_0 + \frac{a^2\hbar c}{2}\left[\frac{6L}{\pi^2\lambda^4} - \frac{1}{\pi\lambda^3} - \frac{2\pi^2}{720L^3} + \ldots\right]$$



$$W_T = W(L) + W(L_0 - L) =$$

$$= \frac{a^2\hbar c}{2}\Bigg[\overbrace{\frac{6L}{\pi^2\lambda^4} + \frac{6(L_0-L)}{\pi^2\lambda^4} - \frac{2}{\pi\lambda^3}}^{\text{indep. of } L} - \frac{2\pi^2}{720}\left(\frac{1}{L^3} + \frac{1}{(L_0-L)^3}\right) + \ldots\Bigg]$$

10^{-5} mbar @ $L = 1\,\mu$m

λ independent potential energy

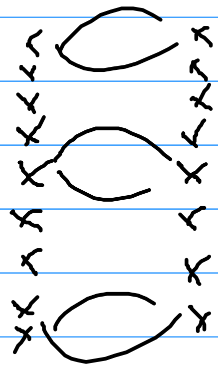
$$U(L) = -\frac{\pi^2\hbar c}{720}\, a^2 \frac{1}{L^3}$$

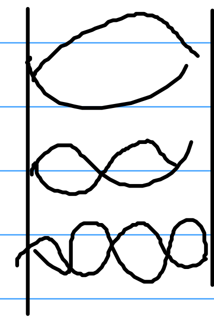
[Casimir energy]

$$P_{\text{vacuum}} = \frac{1}{a^2}\frac{\partial U}{\partial L} = \frac{\pi^2\hbar c}{240}\frac{1}{L^4}$$

Discussion:

Two ways to derive Casimir forces





$$\sum \frac{\hbar\omega}{2}$$

See paper R. Jaffe, PRD

- both views are completely equivalent
- Casimir is independent of atomic properties in the limit $\alpha \to \infty$

[$\alpha \to 0$, $a_0 \to \infty$, Casimir force $\to 0$]

For $d \leq 0.5\,\mu m$ $\qquad (\alpha = 1/137)$

$\alpha \gtrsim 10^{-5}$