DRESSED ATOMS

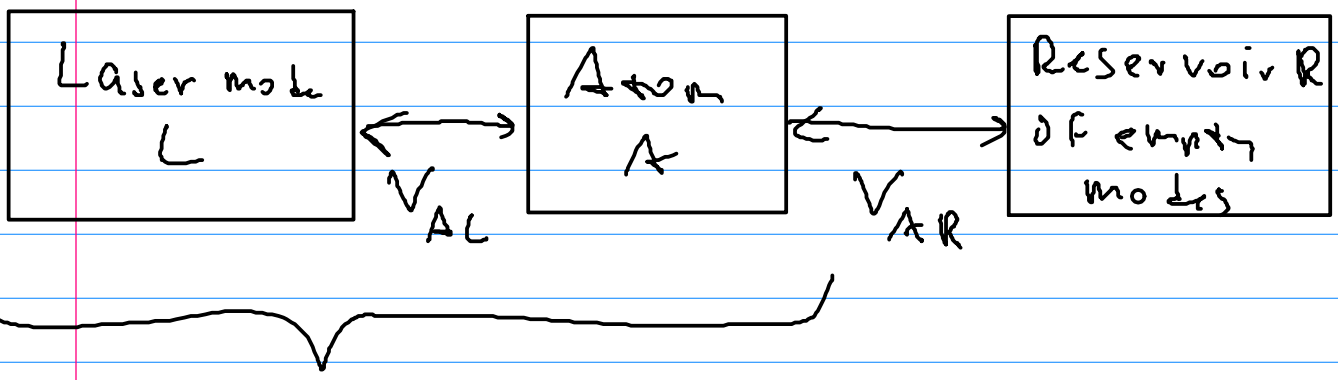
API, CH. VI

= an atom + ONE mode of the e.m. Field

"Nothing new" beyond OBE, BUT: new insight

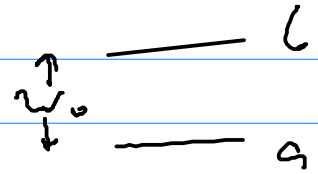
"Difference" to the OBE

- combined state of atom + em Field
- Change of basis: dressed basis



$$H_L = \hbar\omega_L\left(a^+ a + \frac{1}{2}\right)$$

$$H_A = \hbar\omega_0\left(|b\rangle\langle b|\right)$$



eigenstates of H_A and H_L

N photons
$|a\rangle$, $|b\rangle$

$\delta_L \ll \omega_0$

$$\left\{|a, N+1\rangle, \; |b, N\rangle\right\} = \mathcal{E}(N)$$

manifold

δ_L ↓ ——— $|a, N+1\rangle$ $\Sigma(N)$

↑ ——— $|b, N\rangle$

——— $|a, N\rangle$

——— $|b, N-1\rangle$ $\Sigma(N-1)$

Atom-Laser Coupling

$$V_{AL} = -\vec{d}\,\vec{E}_\perp(R)$$

$\vec{E}_\perp$ — Laser Field operator

$$\vec{E}_\perp(\vec{R}) = \sqrt{\frac{\hbar\omega_L}{2\varepsilon_0 V}}\,\hat{\varepsilon}_L\,(a + a^+)$$

$$\vec{d} = \vec{d}_{ab}\,(|b\rangle\langle a| + |a\rangle\langle b|)$$

coupling constant $g = -\hat{\varepsilon}_L \vec{d}_{ab}\sqrt{\quad}$

Coupling within each manifold $\Sigma(N)$

$$\langle b, N | V_{AL} | a, N+1\rangle = g\sqrt{N+1}$$

$a, N+1$ also couples to $b, N+2$

b, N $a, N-1$

nonresonant

neglected within the RWA

N is large, then $\Delta N \sim \sqrt{N} \ll N$

$\Rightarrow \nu_N \simeq g\sqrt{\langle N \rangle}$ independent of N

$$\vec{\mathcal{E}}_0 = 2\vec{\mathcal{E}}_L \sqrt{\frac{\hbar\omega_L}{2\varepsilon_0 V}}\sqrt{\langle N \rangle}$$

$\hbar\Omega_1 := -\vec{d}_{AB}\cdot\vec{\mathcal{E}}_0 \Rightarrow \nu_N = \hbar\Omega_1/2$

Rabi Frequency Ω_1

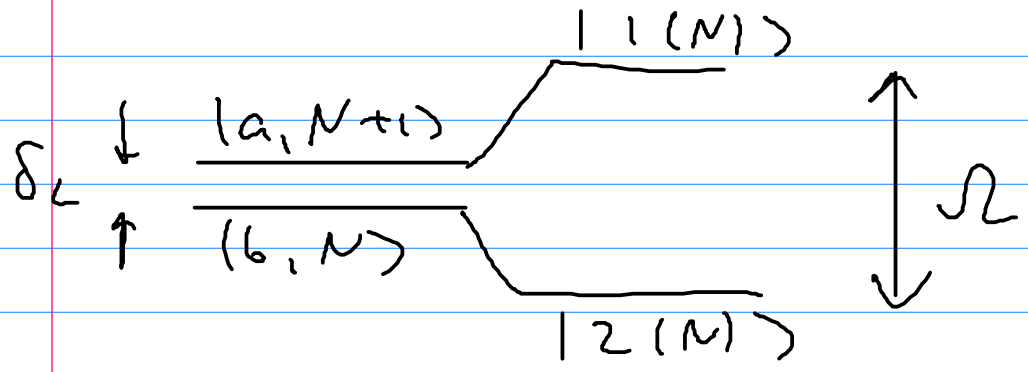
Hamiltonian for each $\mathcal{E}(N)$

$$\frac{\hbar}{2}\begin{pmatrix} -\delta_L & \Omega_1 \\ \Omega_1 & \delta_L \end{pmatrix}$$

coupled pendulum Hamiltonian

Solution $\hbar\Omega = \hbar\sqrt{\delta_L^2 + \Omega_1^2}$

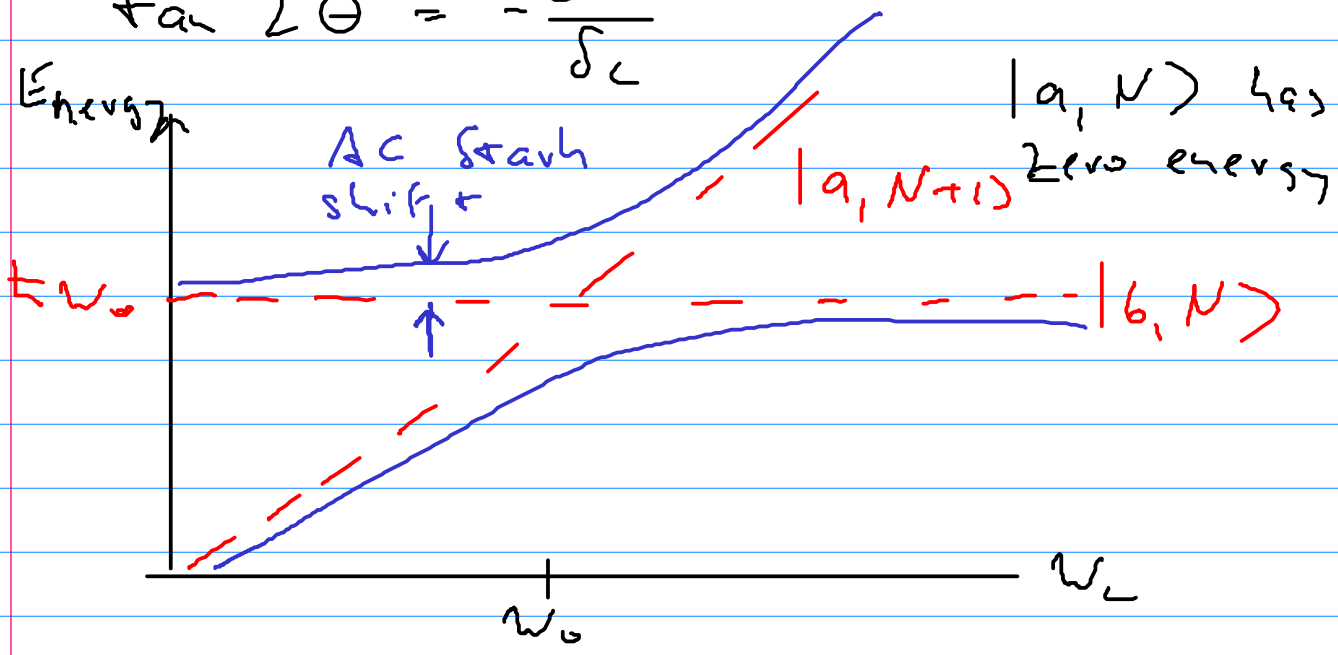
generalized Rabi Frequency



$$|1(N)\rangle = \sin\theta\,|a, N+1\rangle + \cos\theta\,|b, N\rangle$$

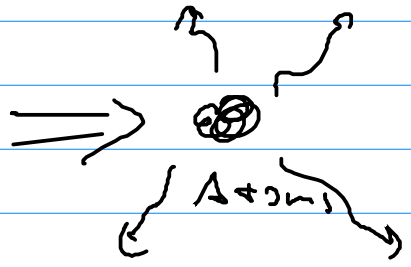
$$|2(N)\rangle = \cos\theta\,|a, N+1\rangle - \sin\theta\,|b, N\rangle$$

$$\tan 2\theta = -\frac{\Omega_1}{\delta_L}$$



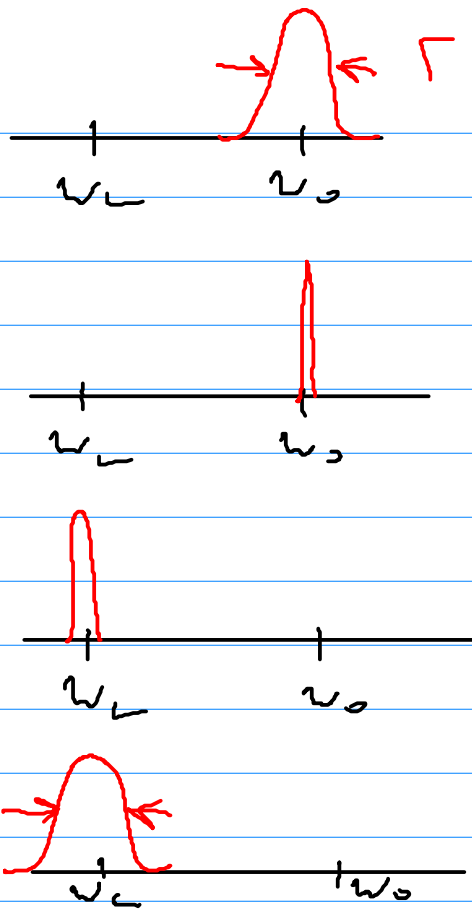
Spectrum of the emitted light

Q. Laser light ω_L
What is the spectrum of the scattered radiation?



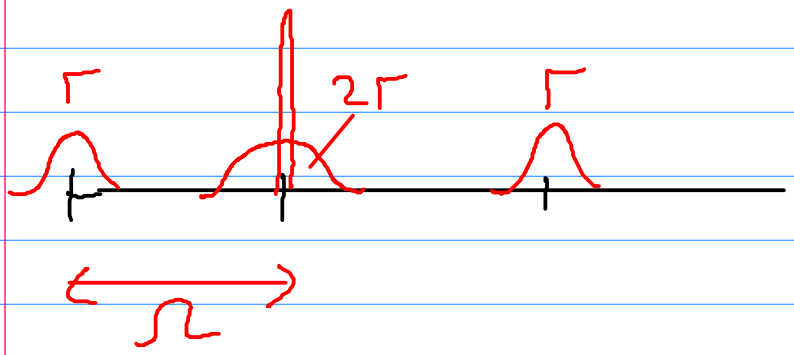
$M = \infty$
no motion

4 options:



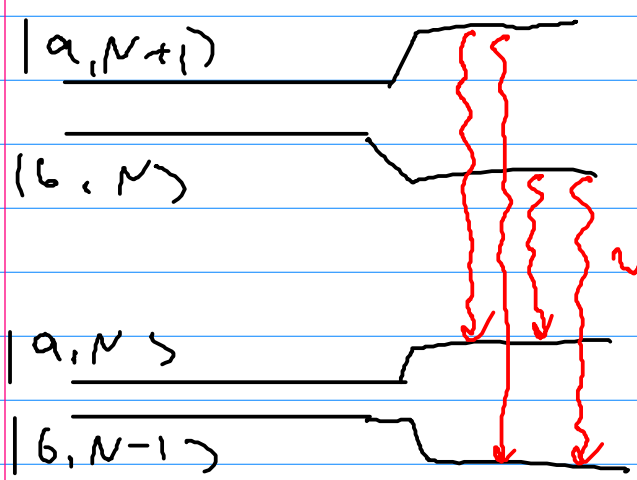
⟸ Only this choice is compatible with energy conservation

Dressed atom



Carrier + two sidebands

Mollow triplet



$\sin\theta\,|a, N+1\rangle + \cos\theta\,|b, N\rangle$

$\cos\theta\,|a, N+1\rangle - \sin\theta\,|b, N\rangle$

$\sin\theta\,|a, N\rangle + \cos\theta\,|b, N-1\rangle$

$\cos\theta\,|a, N\rangle - \sin\theta\,|b, N-1\rangle$

Fluorescence triplet

ω	Rate
$\omega_L + \Omega$	$\Gamma_{1\to 2} = \Gamma \cos^4\theta$
$\omega_L - \Omega$	$\Gamma_{2\to 1} = \Gamma \sin^4\theta$
ω_L	$2\Gamma \cos^2\theta \sin^2\theta$

Population (Intensity = population × rate)

Rate equation

$$\dot{\pi}_1 = -\pi_1 \Gamma_{1\to 2} + \pi_2 \Gamma_{2\to 1} \quad \text{etc}$$

$$= 0 \quad \text{in} \quad \pi_1^{st} \quad \text{steady state}$$

$$\pi_1^{st} = \frac{\Gamma_{2\to 1}}{\Gamma_{1\to 2} + \Gamma_{2\to 1}} = \frac{\sin^4\theta}{\cos^4\theta + \sin^4\theta}$$

$$\dot{\pi}_1 = -\Gamma_{Pop}\left(\pi_i - \pi_1^{st}\right)$$

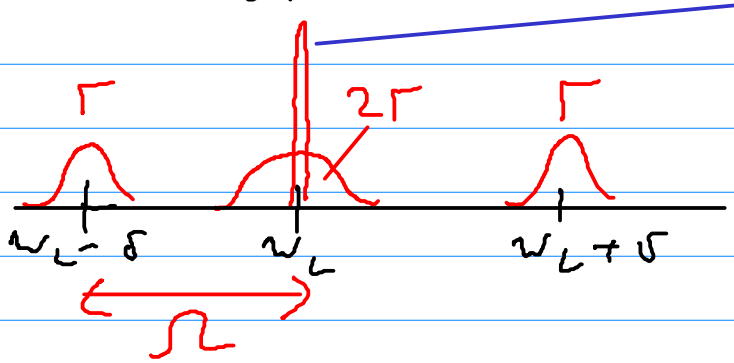
$$\Gamma_{1\to 2} + \Gamma_{2\to 1}$$

Relaxation rate for populations

Similar: Γ_{col}

Relaxation rate for coherence between dressed states

Dressed atom



coherent peak

$I_{coh} \to 100\%$ $\delta \to 0$

$\to 0\%$ $\delta \to \infty$

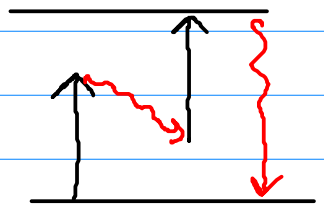
Width of emission lines

- Central feature width Γ_{pop}
- Sidebands Γ_{coh}

OBE - linear matrix equation, complex eigenvalues r_i

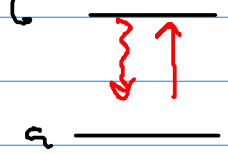
$\delta_L \gg \Omega_1, \Gamma$	$\omega_L = \omega_0$ $\Omega_1 \ll \Gamma$	$\Omega_1 \gg \Gamma, \omega_L = \omega_0$
r_i $i\delta_L + \Gamma/2$	$\Gamma/2$	$i\Omega_1 + \frac{3}{4}\Gamma$
$-i\delta_L + \Gamma/2$	$\Gamma/2$	$i\Omega_1 + \frac{3}{4}\Gamma$
Γ	Γ	$\Gamma/2$

Diagrams



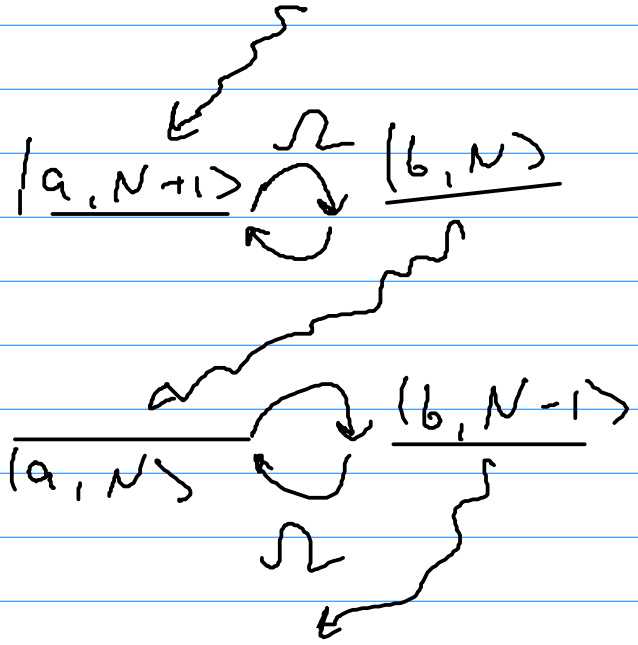
lower and upper sideband

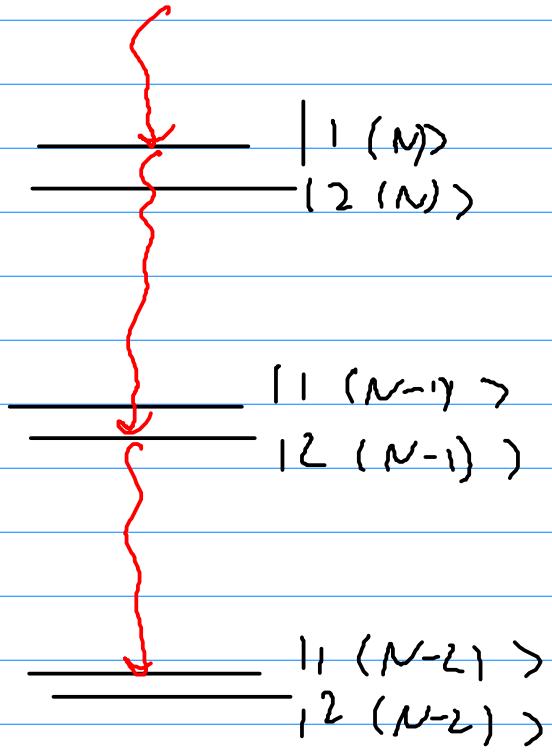
inelastic component of the central peak



inverse scattering of the excited state $|b\rangle$

Radiative cascade in the uncoupled and dressed state basis





anti-bunching

temporal resolution better Ω^{-1} ⇒ can't see sidebands

$\omega_L \pm \Omega$ photons alternate