Dipole Forces within the Dressed-Atom Picture

Ref: • J. Dalibard, C. C.-T., JOSA B 1985
• API

Provides insight for
- Dipole traps
- Blue molasses

Remember:

- Splitting between dressed states

$$\hbar\,\Omega(r) = \hbar\sqrt{\Omega_1^2(r) + \delta_L^2}$$

- Steady-state populations of dressed states

$$\Pi_1^{st} = \frac{\sin^4\theta}{\cos^4\theta + \sin^4\theta} \qquad \left(\tan 2\theta = -\Omega_1/\delta_L\right)$$

$$\Pi_2^{st} = \frac{\cos^4\theta}{\sin^4\theta + \cos^4\theta}$$

- Relaxation rate for $\Pi_{1,2}$ to reach steady state

$$\Gamma_{pop} = \Gamma\left(\cos^4\theta + \sin^4\theta\right)$$

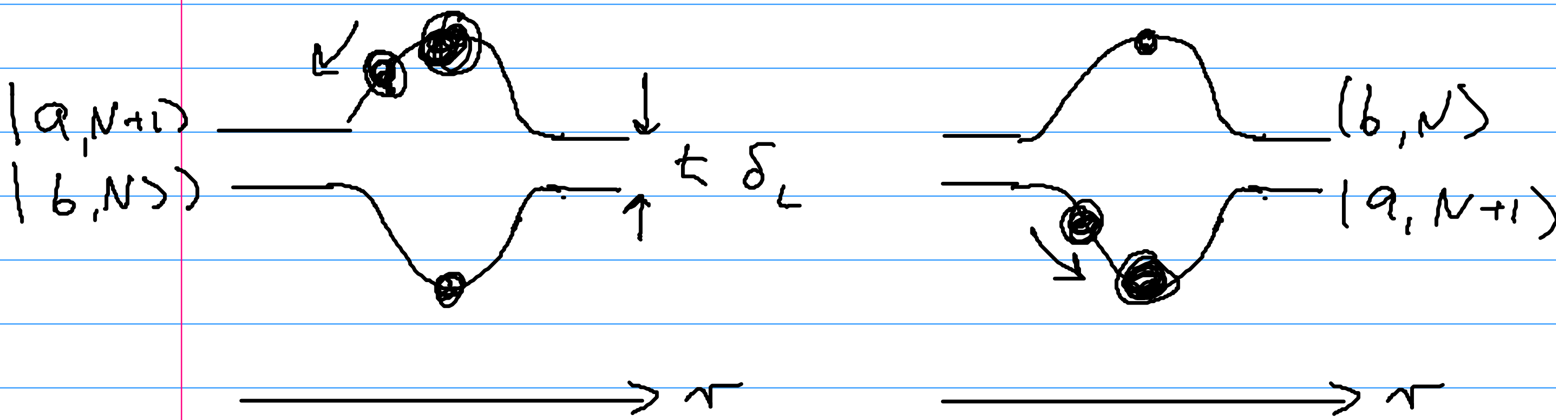
Mean dipole force, $v=0$

$$F_1 = -\frac{\hbar}{2}\nabla\Omega(r)$$

Force = derivative of position-dependent energy level

$$F_2 = +\frac{\hbar}{2}\nabla\Omega(r)$$

$$\langle F_{dip}\rangle = F_1\Pi_1^{st} + F_2\Pi_2^{st} = -\frac{\hbar\delta_L}{2}\frac{\Omega_1^2}{\frac{\Omega_1^2}{2}+\delta_L^2}\,\vec{\alpha}$$



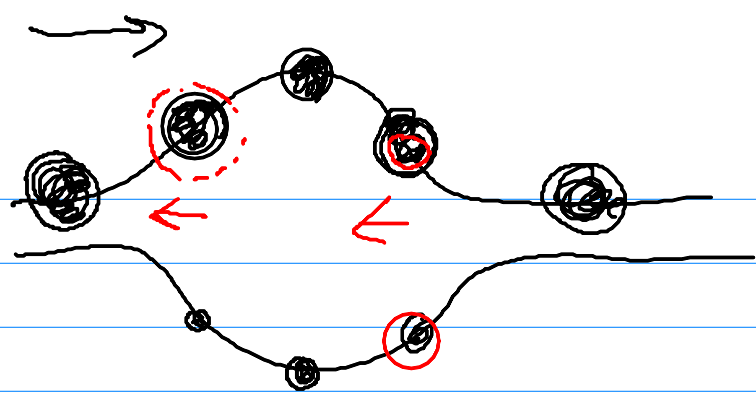
$\delta_L > 0$ $\delta_L < 0$

$\delta_L = 0$ $\Pi_1^{st} = \Pi_2^{st} \Rightarrow \langle F_{dip}\rangle = 0$

Mean dipole force for a slowly moving atom

$$\langle F_{dip}\rangle = F_1\pi_1 + F_2\Pi_2$$

time lag by $\tau_{pop} = 1/\Gamma_{pop}$



$$\Pi_i = \Pi_i^{st}\left(\vec{r} - \vec{v}\,\tau_{pop}\right)$$

Extra force due to time lag always opposes $\vec{v}$ $\Rightarrow$ Friction

$\delta_L > 0$

$$\vec{F}_{dip}(\vec{r}, v) = \underbrace{\vec{F}_{dip}^{st}}_{v=0} - \frac{2\hbar\delta_L}{\Gamma}\left(\frac{\Omega_1^2(r)}{\Omega_1^2(r) + 2\delta_L^2}\right)(\vec{\alpha}\cdot\vec{v})\,\vec{\alpha}$$

Standing wave $\quad \Omega_1(x) = 2\Omega_1 \cos kx$

λ-average $\quad \langle F_{dip}\rangle = -\alpha v$

For $\delta_L \gg \Omega_1$, note that $\vec{\alpha} \simeq \vec{k}$

$$\alpha \simeq \frac{\hbar\delta_L}{\Gamma}\frac{\Omega_1^6}{\delta_L^6}k^2$$

We will later derive this again

Atomic momentum diffusion

$$2D_p = \frac{d}{dt}\left(\langle p^2\rangle - \langle p\rangle^2\right)$$

$$= 2\langle\vec{p}\cdot\vec{F}(0)\rangle - \langle\vec{p}\rangle\langle\vec{F}(0)\rangle =$$

$$2\int_{-\infty}^{0}\left(\langle\vec{F}(t)\vec{F}(0)\rangle - \langle\vec{F}(t)\rangle\langle\vec{F}(0)\rangle\right)d\tau$$

$$D_p = \int_0^\infty d\tau \left[\langle F(0) F(\tau) \rangle - \langle \vec{F}(0) \rangle^2 \right]$$

Momentum diffusion is caused by fluctuations of the force

$$\langle \dot{E}_{heat} \rangle = D_p / M \; ; \quad k_B T = D_p / \alpha$$

Radiative cascade:
Force switches between F_1 and $F_2 = -F_1$

$$|F_1| = |F_2| = \frac{\hbar}{2} \nabla \Omega_1$$

On resonance: correlation time $\tau = 2/\Gamma$

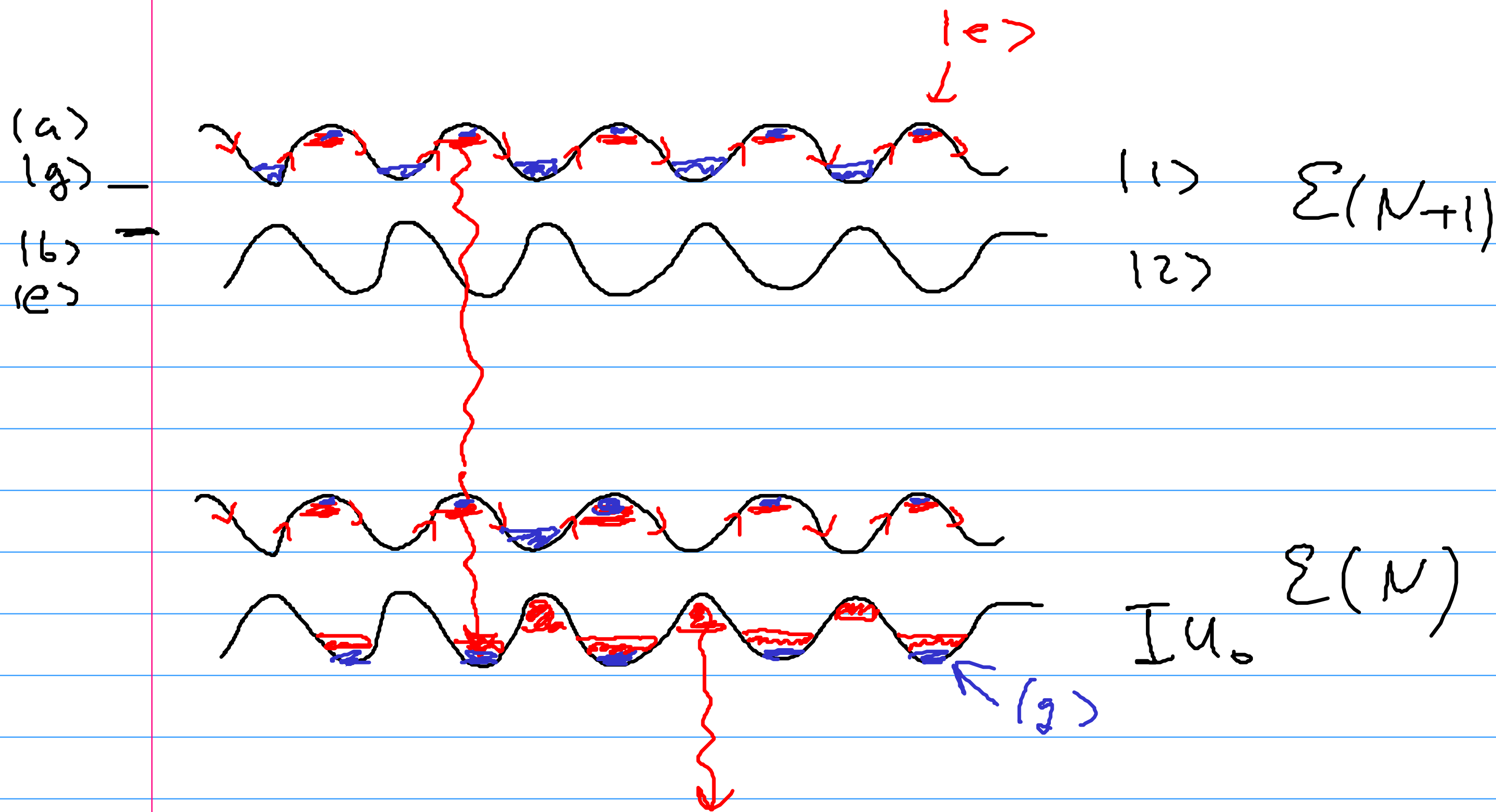
$$D_{dip} \simeq \frac{\hbar^2 (\nabla \Omega_1)^2}{2\Gamma}$$

result for arbitrary detuning δ_L

$$D_{dip} \simeq \frac{\hbar^2 (\nabla \Omega_1)^2}{2\Gamma} \left(\frac{\Omega_1^2}{\Omega_1^2 + 2\delta^2} \right)^3$$

Atoms moving in a standing wave

$k v \gtrsim \Gamma$ — atoms moves several wavelengths per lifetime



both |1⟩ |2⟩ ; preferential spont. em at the top of the hill

⇒ SISYPHUS cooling

$$\text{Cooling rate} \approx U_0 \Gamma_{1\to 2} \Pi_1$$

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Cooling Atoms with Stimulated Emission

First experimental demonstration

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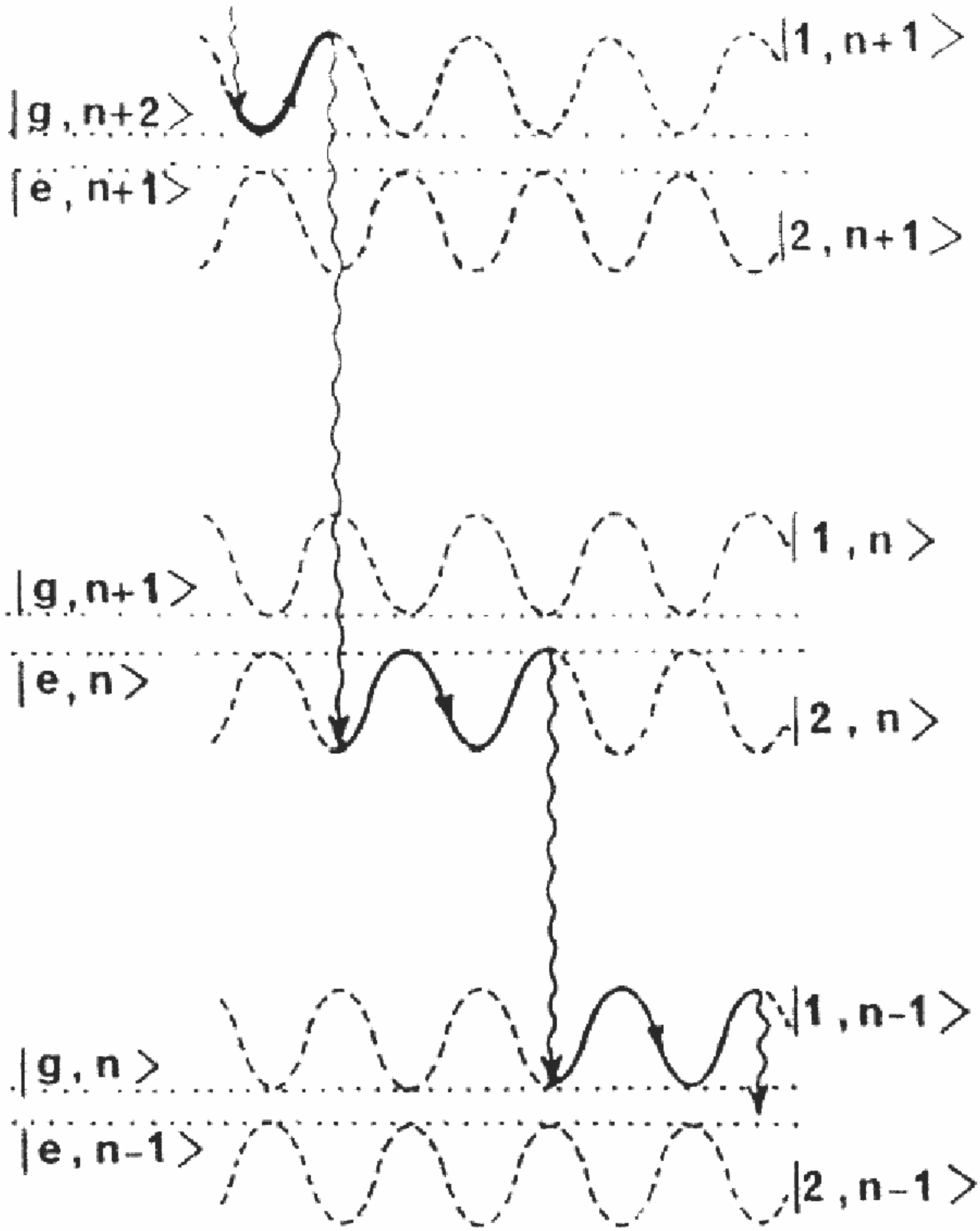


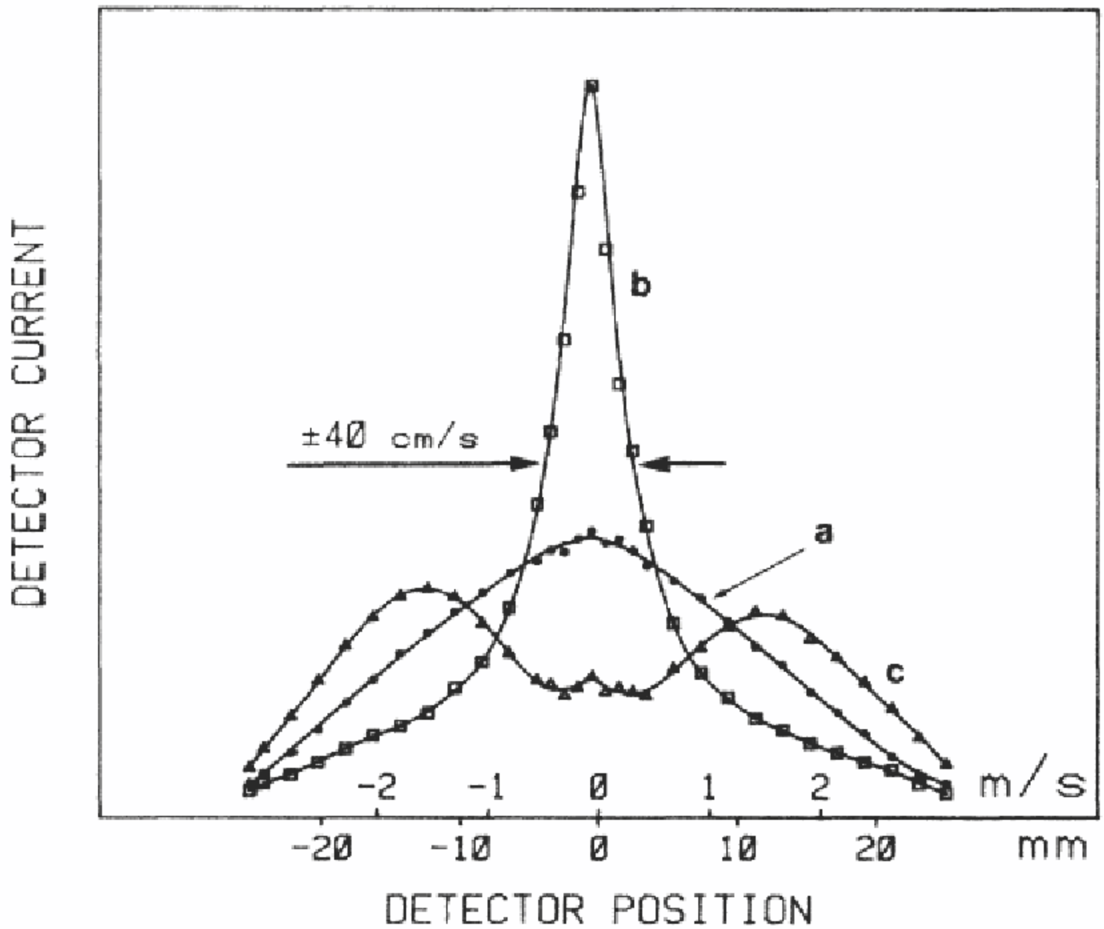


FIG. 3. Detector current vs position of the hot-wire detector. The corresponding transverse atomic velocities are given in m/s. Peak current is 2.2×10^9 atoms/s. The full lines are intended merely as visual aids. Curve *a*, laser beam off (HWHM 2 m/s); curve *b*, laser beam on with a positive detuning ($\delta/2\pi=+30$ MHz); curve *c*, laser beam on with a negative detuning ($\delta/2\pi=-30$ MHz).

Cooling in a standing wave

Summary of concepts in the simplest limit $\delta_L \gg \Omega_1 \gg \Gamma$

neglecting factors on the order of unity

$\hbar = 1$

$\Omega_1(x) = 2\Omega_1 \cos kx$ Standing wave

$$|1\rangle = |a\rangle + \frac{\Omega_1(x)}{\delta_L}|b\rangle$$

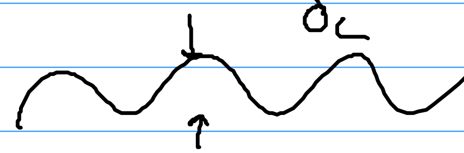
$$|2\rangle = |b\rangle - \frac{\Omega_1(x)}{\delta_L}|a\rangle$$

$$\Gamma_{1\to 2} = \Gamma\left(\frac{\Omega_1(x)}{\delta_L}\right)^4$$

$$\Gamma_{2\to 1} = \Gamma$$

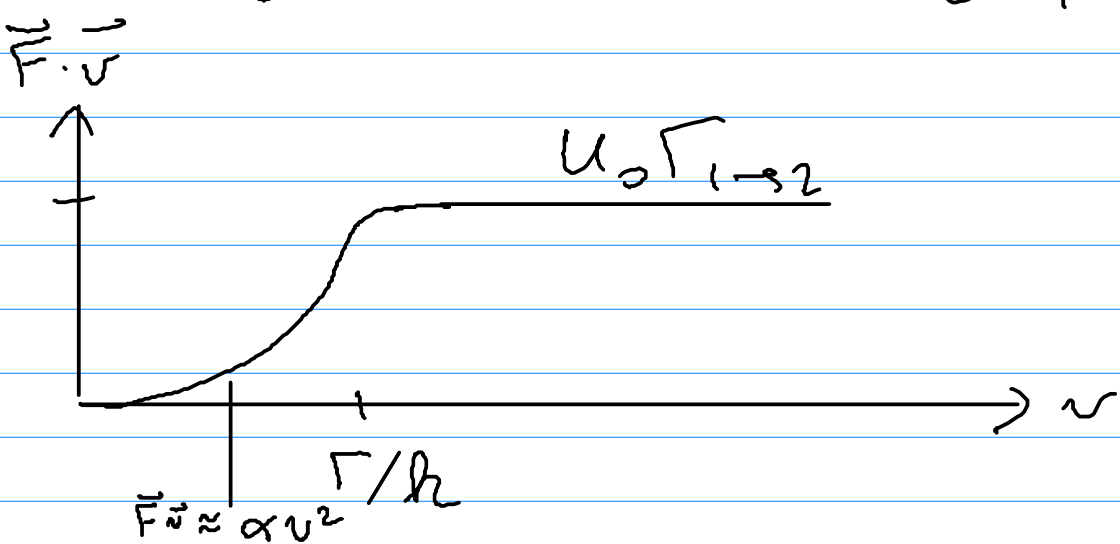
$$U_0 = \frac{\Omega_1^2}{\delta_L}$$

Dressed state potential



$v \geq \Gamma/k$ Sisyphus cooling

Cooling rate: $\vec{F}\cdot\vec{v} = U_0\,\Gamma_{1\to 2}$



$$\text{estimate } \alpha = \lim_{v \to 0} \frac{F(v)}{v}$$

$$\simeq \frac{U_0 \Gamma_{1\to 2}}{(\Gamma/k)^2}$$

Same as obtained before

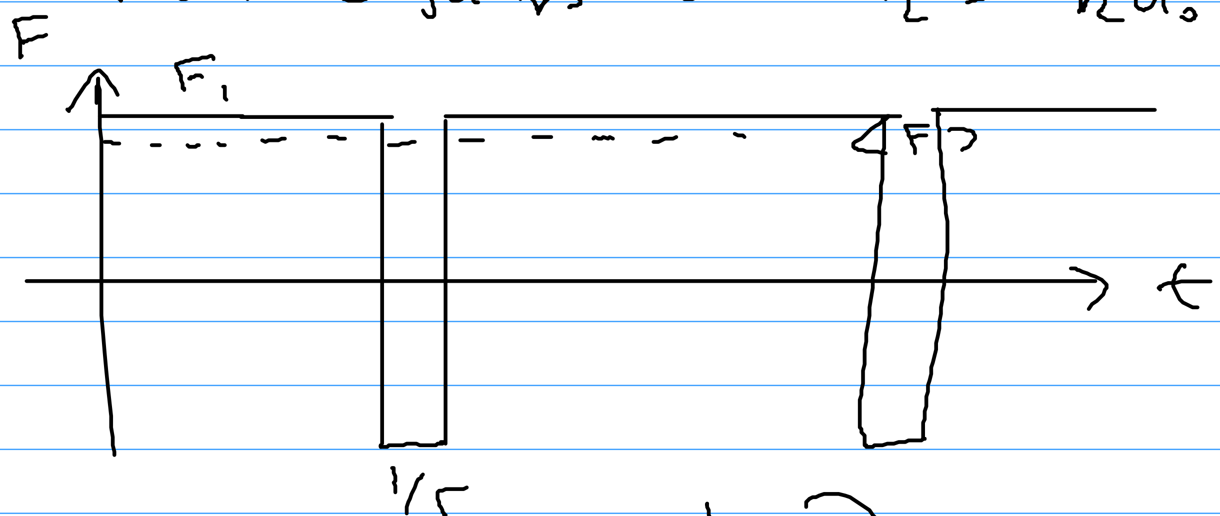
$$= \frac{\Omega_1^2}{\delta} \frac{\Gamma \Omega_1^4}{\delta_L^4} \frac{k^2}{\Gamma^2} = \frac{\Omega_1^6}{\delta_L^5} \frac{k^2}{\Gamma}$$

Diffusion

$$D = \int \langle \Delta F(0) \, \Delta F(t) \rangle \, d\tau$$

atoms mainly in state $|1\rangle$, $F_1 = -\nabla U_1(r) \simeq k U_0$

For a duration $1/\Gamma$ at a rate $\Gamma_{1\to 2}$ the force jumps to $-\nabla U_2 = -k U_0$



$$D = \underbrace{(k U_0)^2}_{(\text{Force})^2} \underbrace{/\Gamma}_{\text{corr. time}} \quad \left. \frac{\frac{1}{\Gamma}}{\frac{1}{\Gamma_{1\to 2}}} \right\}$$

Probability that the atom is in $|2\rangle$ in our ensemble

$$= \hbar^2 u_0^2 \Gamma_{1\to 2} / \Gamma^2$$

Ultimate temperature:

$$k_B T = D/\alpha = U_0$$

i.e. Cooling in a SW cannot localize a two-level atom in the potential minimum

Note: For $U_0 < \Gamma$, we have to include heating by atomic recoil.

As a result, blue molasses cannot cool to temperatures lower than the Doppler limit, $\Gamma/2$.

Discussion

- Dipole traps
- Electric & magnetic forces
- Energy conservation

Dipole $\begin{cases} \text{traps} \\ \text{potential} \end{cases}$ $$U = \frac{\hbar\delta}{2} \log\left[1 + \frac{I/I_0}{1+(2\delta/\Gamma)^2}\right]$$

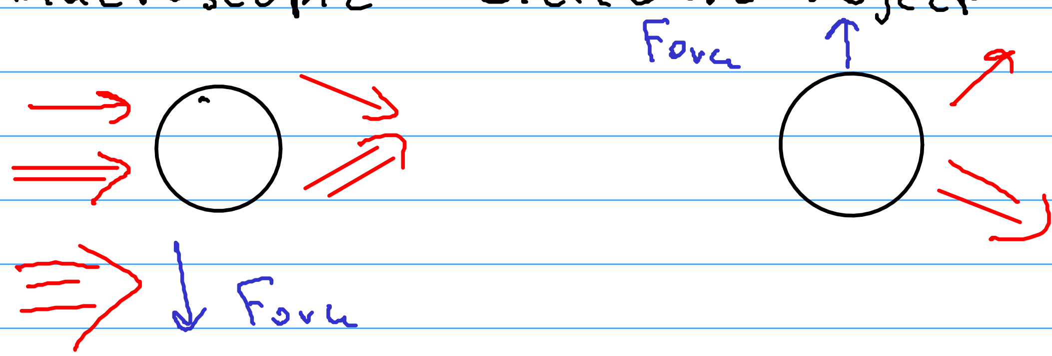
explanations:

- OBE
- dressed atom picture
- $W = -\vec{d}\,\bar{E}$

$\begin{matrix} \text{below} \\ \text{above} \end{matrix}\Big\}$ resonance $\vec{d}$ $\begin{matrix} \text{in} \\ \text{out of} \end{matrix}\Big\}$ phase with E

$\Rightarrow \begin{cases} \text{attractive} \\ \text{repulsive} \end{cases}$ Forces

- macroscopic dielectric object



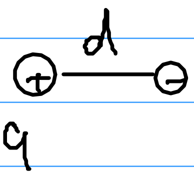
$n > 1$ $\qquad\qquad$ $n < 1$

optical tweezers $\Rightarrow$ manipulate objects within a cell

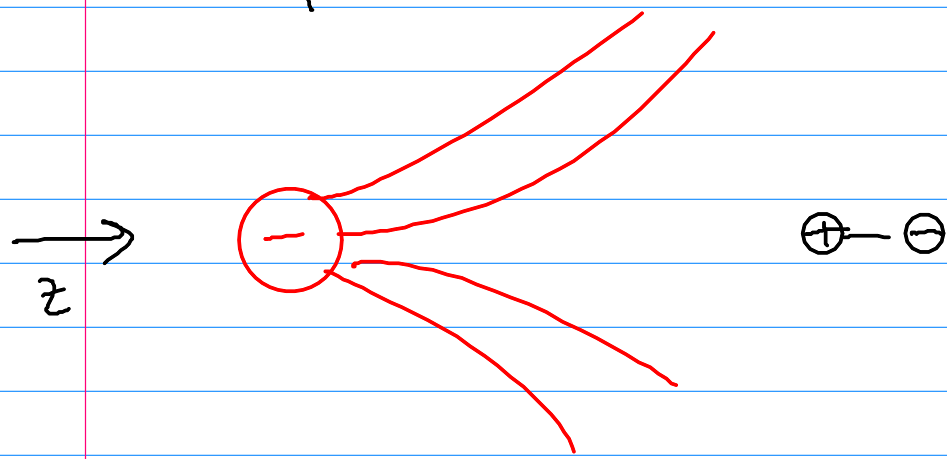
revolution in biology (Ashkin, Chu)

Is the stimulated force an electric force, a magnetic force, or both?

First electrostatics

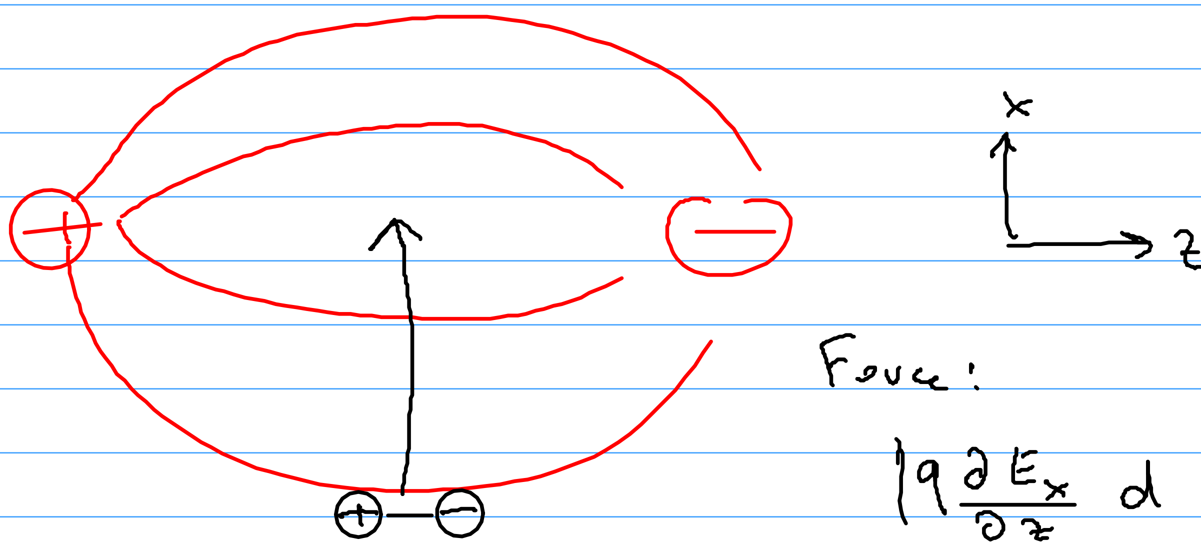


$q d = \alpha E$ energy $\frac{1}{2}\alpha E^2$



Force $q \frac{\partial E_z}{\partial z} d$

Work $q d E_z \times \frac{1}{2}$



Force:

$\left| q \frac{\partial E_x}{\partial z} d \right|$

$= \left| q \frac{\partial E_z}{\partial x} d \right|$

IF $\vec{\nabla} \times \vec{E} = 0$

Work $= \int$ Force $dx = \vec{E} \cdot \vec{p} \times \frac{1}{2}$

Now: oscillating dipole and SW

energy $\frac{1}{2}\alpha E^2$

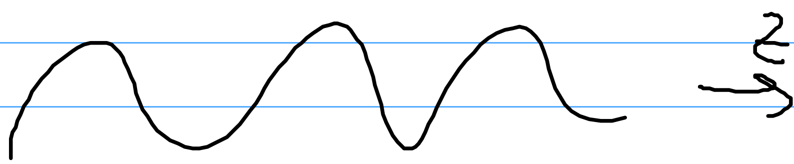
What is the force?

Plane SW el. Force $= 0$

$\vec{p} \perp \vec{\nabla} E$

We show below:

$$F_{Lorentz} = \frac{d}{dz}\left(\frac{1}{2}\alpha E^2\right)$$



SW $\vec{E} = A_0 \cos kz \sin\omega t \, \hat{e}_x$

$\vec{B} = -A_0 \sin kz \cos\omega t \, \hat{e}_y$

Dipole potential $\frac{1}{2}\alpha \overline{E^2} = \frac{1}{4}\alpha A_0^2 \cos^2 kz$

Dipole $p = \alpha E$

Lorentz force $\quad q \frac{\vec{v}}{c} \times \vec{B} \longrightarrow \frac{\dot{\vec{p}}}{c} \times \vec{B}$

Force $F_{Lorentz} =$

$$\frac{1}{c} \alpha A_0 \cos kz \cos \omega t \; \omega \left(- A_0 \sin kz \cos \omega t \right)$$

$$\overline{F_L} = -\frac{1}{2} \alpha A_0^2 k \cos kz \sin kz$$

$$= \frac{d}{dz} \left(\frac{1}{4} A_0^2 \cos^2 kz \right) = \frac{d}{dz} \left(\underbrace{\frac{1}{2} \alpha \overline{E^2}}_{\text{dipole potential}} \right)$$

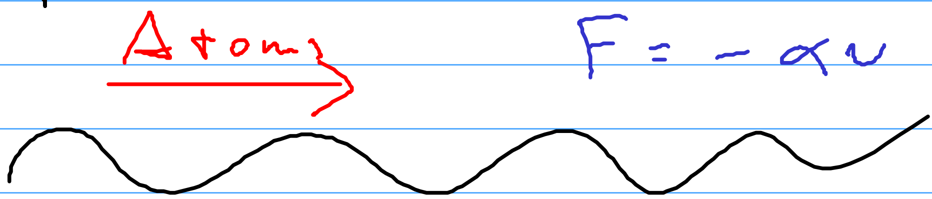
$\Rightarrow$ stimulated light force can be either electrical ($p_z \nabla E_z$) or magnetic ($\dot{p} \times B$)

each of the two forces may not be curl free, but the sum of both !

Energy conservation & dipole force

how can you exchange energy by redistribution of photons?

- cw experiment



$F = -\alpha v$

BLUE MOLASSES

Where does the K.E. of the atom go?
Sideband emission
upper sideband is emitted at larger Ω than the lower sideband
(see Sisyphus picture)

- transient experiment

atom is dielectric medium
$\Rightarrow$ phase modulation and frequency modulation of transmitted light

$\Rightarrow$ When atoms gain K.E., the transmitted photons are on average red-shifted.

No spontaneous emission necessary in this case for energy conservation.