Applications of the spontaneous Force

- Molasses
- Beam slowing
- Magneto-Optical trap

$$F_{diss} = \hbar k \frac{\Gamma}{2} \frac{I/I_0}{1 + I/I_0 + \left(\frac{2(\delta + kv)}{\Gamma}\right)^2}$$

$$I/I_0 = 2\Omega_1^2/\Gamma^2$$

I_0 Saturation intensity

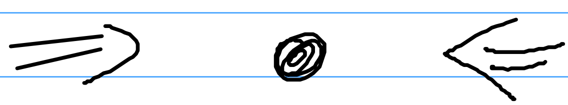
Na-D $12\,mW/cm^2$

$$a_{max} = \frac{F}{m}\Big|_{max} \simeq 10^5 g$$

stops 1000 m/s Na in 1 ms or 0.5 m

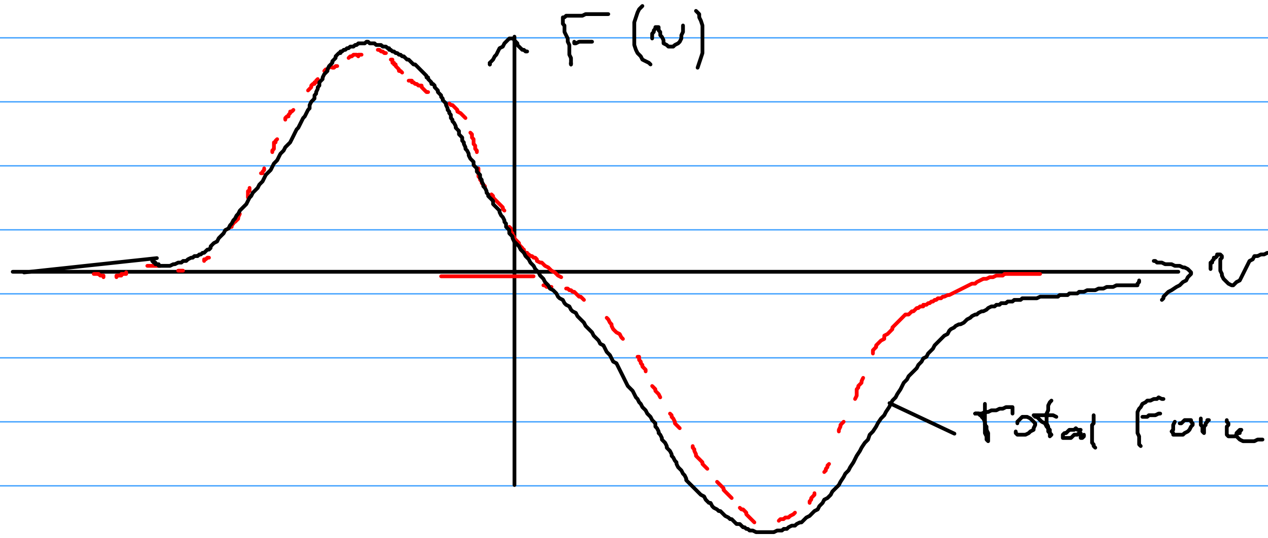
≙ to the Force of $\frac{1\,mV}{cm}$ on Na^+

$\boxed{\text{Optical molasses}}$



Assumption: total Force = sum of two forces
ignores standing wave effects
valid at low intensity $I \ll I_0$

$$F(v) = \hbar k \frac{\Gamma}{2} \frac{I}{I_0} \left(\frac{1}{1 + \left(\frac{2(\delta - kv)}{\Gamma}\right)^2} - \frac{1}{1 + \left(\frac{2(\delta + kv)}{\Gamma}\right)^2} \right)$$



$$F(v) \xrightarrow[v \ll \Gamma/k]{} -\alpha v$$

damping

"optical molasses"

$$\alpha = 2\hbar k^2 \frac{(2 I/I_0)(2\delta/\Gamma)}{\left(1 + \left(\frac{2\delta}{\Gamma}\right)^2\right)^2}$$

The Doppler cooling limit

$$F = -\alpha v$$

$$\dot{E}_{cool} = F v = -\alpha v^2 = -\frac{2\alpha}{m} E$$

exponential decay

Heating:

- Spont emission

  random momentum kicks by $\hbar k$

$$p_{\text{Final}}^{\text{RMS}} = \sqrt{N}\,\hbar k$$

$$\langle p^2 \rangle = (\hbar k)^2 N$$

Scattering Rate

$$\Rightarrow \langle \dot{p^2} \rangle_{\text{spont Em}} = (\hbar k)^2 \gamma_s$$

- Fluctuations in absorption

  Poissonian statistics

  variance $\sqrt{N}$

  $\Rightarrow$ Same as for spont em

$$\langle \dot{p^2} \rangle_{\text{Abs}} = (\hbar k)^2 \gamma_s$$

$$\dot{E}_{\text{heat}} = \frac{\langle \dot{p^2} \rangle}{2M} = \frac{2\hbar^2 k^2 \gamma_s}{2M} = \frac{D}{M}$$

Momentum diffusion coefficient

$$D := \frac{\langle \dot{p^2} \rangle}{2}$$

Equilibrate: $\dot{E}_{\text{heat}} = \dot{E}_{\text{cool}}$

$$\frac{D}{M} = \frac{2\alpha}{M} E \Rightarrow 2E = k_B T = \frac{D}{\alpha}$$

1 D

Fluctuation-dissipation theorem

Einstein relation

For $I \ll I_0$

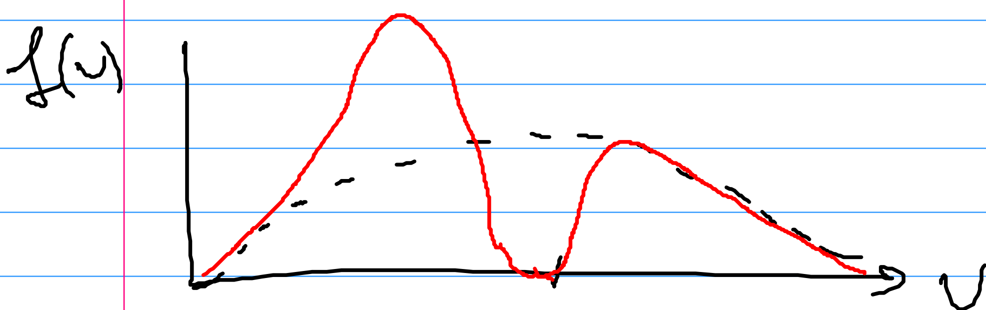
$\delta = -\Gamma/2$

$$k_B T_{DOPPLER} = \frac{\hbar \Gamma}{2}$$

Doppler limit

Na 240 μK
30 cm/s

Beam slowing



Single beam
Fixed frequency

Further slowing: broad range of frequencies
- white light
- diffuse light

chirped slowing:
- make atoms "ride the surf"
- chirp and deceleration of the atoms are synchronized.

Zeeman slowing
- see HW
- CW version of chirped slowing with 100% duty cycle

Note: $a<0$, $v>0$, $k>0$, $a_{max}>0$

Chirped slowing

$$F = \underbrace{-\hbar k \frac{\Gamma}{2}}_{M a_{max}} \frac{I/I_0}{1 + I/I_0 + \left[\frac{2(\delta + k v)}{\Gamma}\right]^2} \qquad (1)$$

① Select deceleration a (<0)

② Determine nominal detuning δ' from

$$a = \frac{-I/I_0}{1 + I/I_0 + \left(\frac{2\delta'}{\Gamma}\right)^2} a_{max} \qquad (2)$$

[has solution if $a < \frac{I/I_0}{1+I/I_0} a_{max}$]

③ Select initial velocity V_0 (cancels out)

$$V(t) := V_0 + a t$$
$$\delta(t) := \delta' - k V(t)$$
$$v' := v - V(t)$$

just definitions

$\delta(t)$ is the laser detuning (chirp)

④ Substitute into (1): $\delta + k v = \delta' + k v'$

$$F = M a_{max} \frac{-I/I_0}{1 + I/I_0 + \left(\frac{2(\delta' + k v')}{\Gamma}\right)^2}$$

⑤ transform to decelerating frame (add fictitious force $-Ma$ from (2))

$$F(v') = M a_{max} \left[\frac{-I/I_0}{1 + I/I_0 + \left[\frac{2(\delta' + k v')}{\Gamma}\right]^2} + \frac{I/I_0}{1 + I/I_0 + \left(\frac{2\delta'}{\Gamma}\right)^2} \right]$$

This is exact (!) for arbitrary v'

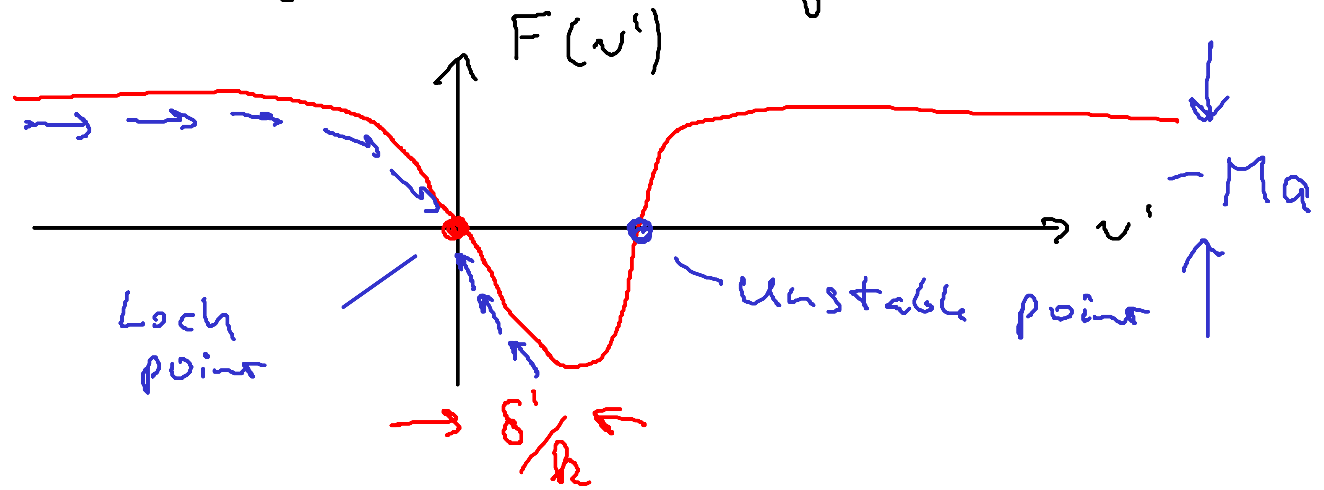
For small v': $F'(v') = -\alpha v'$

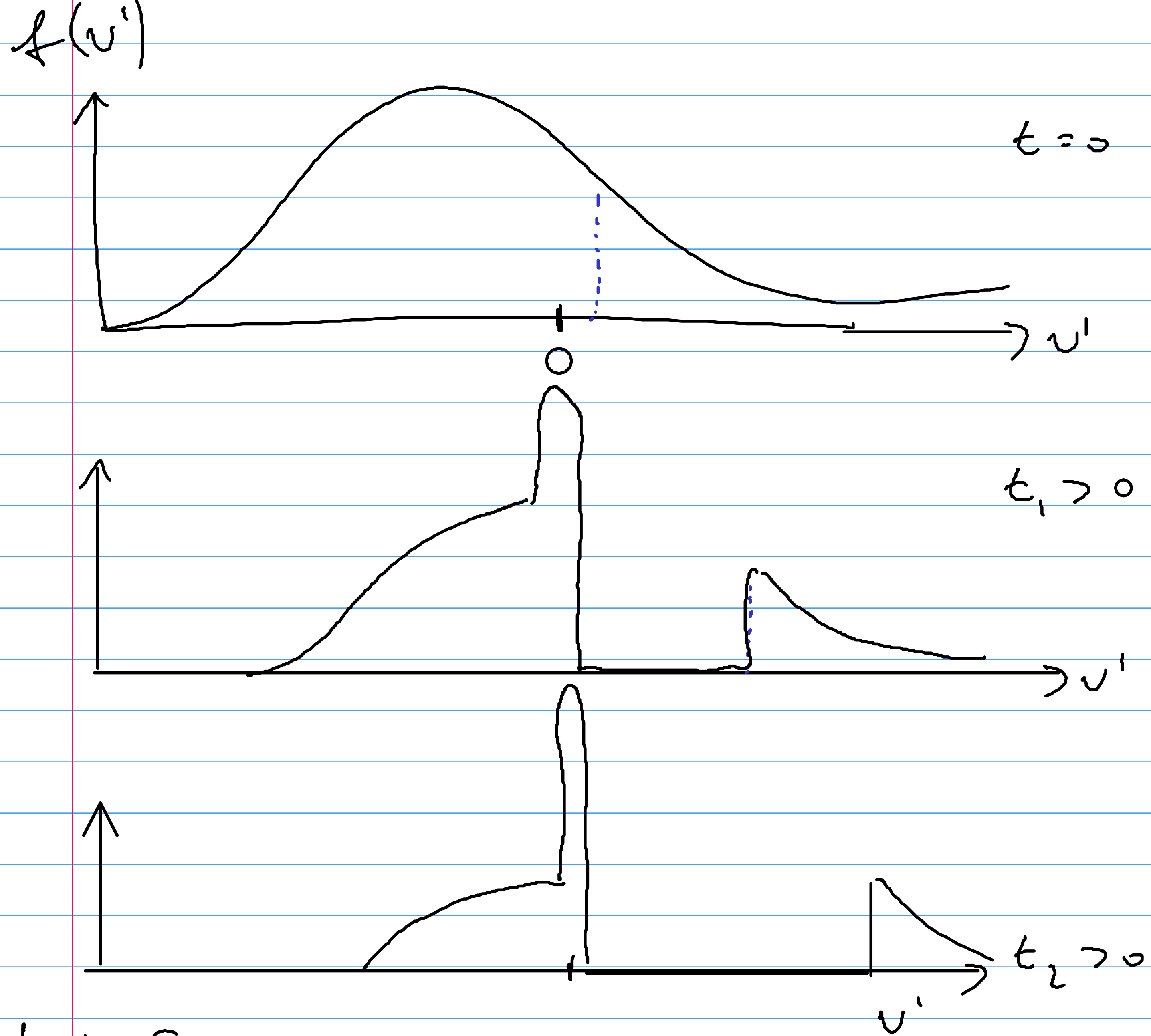
$$\alpha_{Dec} = \frac{1}{2} \alpha_{MOL} \text{ (only one beam)}$$

$$D_{dec} = \frac{1}{2} D_{Mol}$$

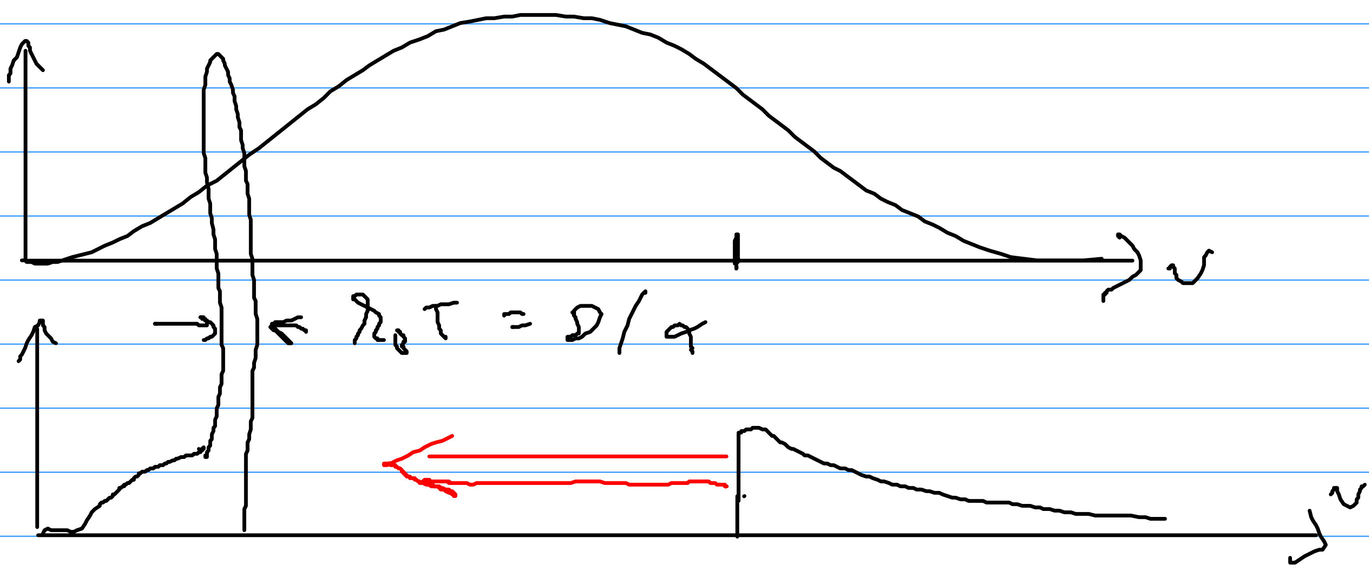
$$\Rightarrow \text{ Same } k T_{min} = k T_{Doppler} = \frac{D}{\alpha}$$

Total force in decelerating frame

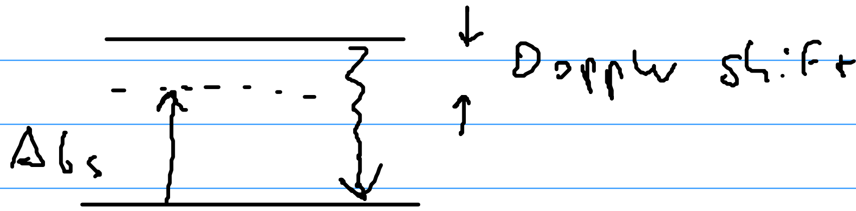




Lab frame



Q: Where does the lost kinetic energy go?



$\Rightarrow$ the emitted light is blue shifted.

• 3D molasses

3×2 beams

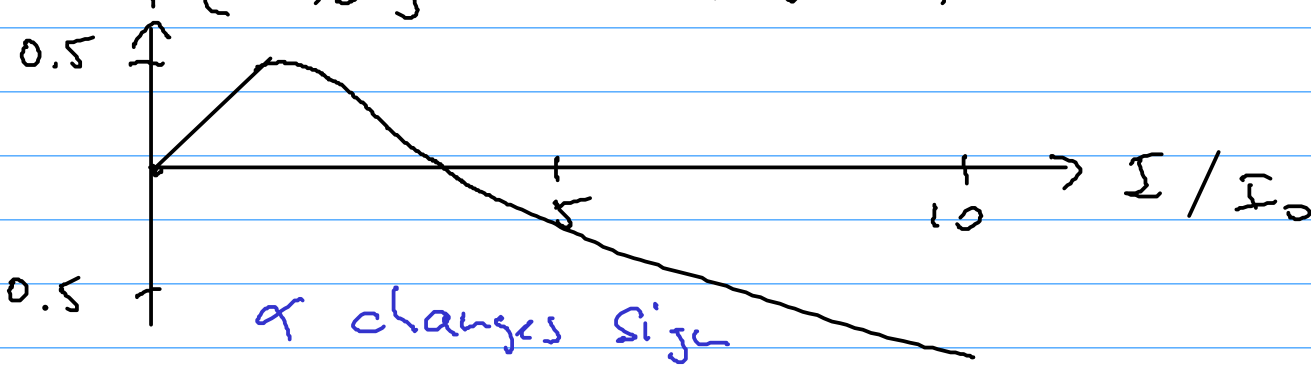
$I \ll I_0$ add up the forces

But: interference of beams leads to polarization gradients, provide additional cooling for multi-level atoms

First observation '85: Chu et al.

• High intensity $I \gtrsim I_0$

$\alpha \, [\hbar k^2]$ Ex: $\delta = -\Gamma$



How to get this from OBE:

$$u = u_{st} + v(\quad)$$

$$u(\vec{r}) \simeq u_{st}(\vec{r} - \vec{v}\,\Delta t)$$

lag time

$$\vec{F} = -\alpha \vec{v} \qquad (\lambda\text{-averaged})$$

Weak intensity $\quad \alpha_{SW} = 2\,\alpha_{TW}$

High intensity $\quad \alpha$ changes sign

The magneto-optical trap (MOT)

History:

Optical Earnshaw theorem (1983)

Phillips, Varenna, p 319

Spontaneous light force traps are NOT possible ☹

Proof: $\quad \vec{F}_s = c\,\vec{S}$ — Poynting vector

$$\text{E\&M:} \quad \cancel{\frac{\partial u}{\partial t}}^{\,0} + \nabla S = 0$$

energy density

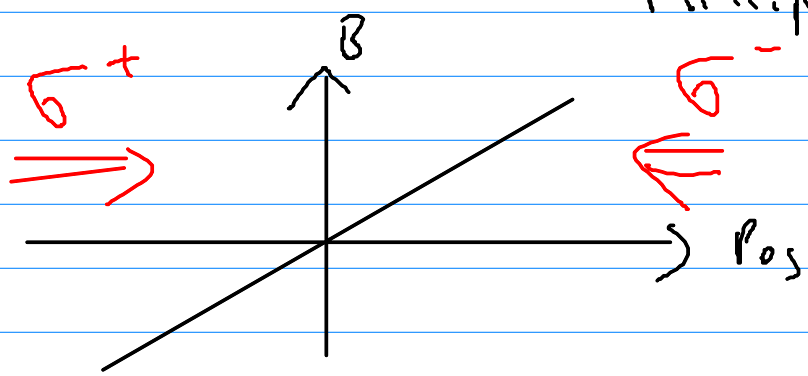
$$\Rightarrow \vec{\nabla} \cdot \vec{F}_s = 0 \; ; \quad \text{trap requires } \vec{\nabla} \cdot \vec{F}_s < 0$$

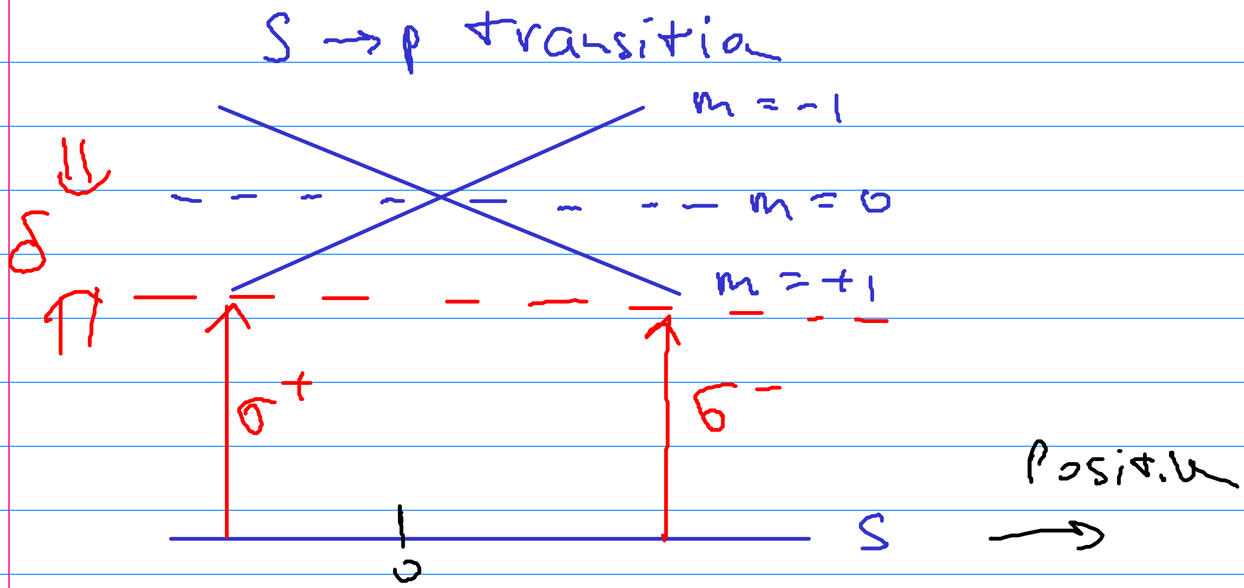
Solution $\nabla \left(\underbrace{c(\vec{r})} \vec{S}(\vec{r}) \right) \neq 0$

$\vec{r}$-dependence: Saturation
optical pumping
Line shifts due to ext. Fields
$\rightarrow$ MOT

MOT

Raab et al 1987
Phillips, Varenna, p. 321





Similar to molasses, but frequency shifts $\pm \beta z$ due to a magnetic field gradient $\beta = \mu B' / \hbar$

$$F = F_R + F_L = \frac{\hbar k \Gamma}{2}\left[\frac{I/I_0}{1+4\left(\frac{\delta - kv - \beta z}{\Gamma}\right)^2} - \ldots\right]$$

For small v, z

$$F(v,z) = (\;)\left[\underbrace{-kv}_{\text{damping}} \underbrace{-\beta z}_{\text{restoring force}}\right]$$

eq. of motion

$$\ddot{z} + \gamma \dot{z} + \omega_{trap}^2 z = 0$$

$$\gamma \frac{\beta}{k}$$

damped HO

typically: overdamped HO

$\omega = 2\pi \; 1\,\mathrm{kHz}$

damping time $50\,\mu s$

static well depth $2\,\mathrm{K}$

dynamic depth is even larger (due to friction force)

Discussion:

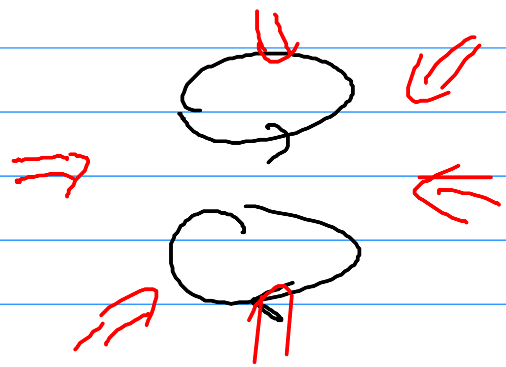
- Multilevel structure
  - Works for HFS
    - combination of optical pumping and Zeeman shifts

• $\gamma_{PG} \gg \gamma_{Doppler}$

Same for restoring force

MOT works even better for multilevel atoms

• Extension to 3D works well



3D B-field gradients: Anti-Helmholtz coils

Vapor Cell MOT (Cs)

T:	$300\,K \rightarrow 3\,\mu K$		$\downarrow 10^8$
n	$10^8/cm^{-3} \rightarrow 10^4$		$\uparrow 10^3$

phase space density D $\uparrow 10^{15}$

$n/T^{3/2}$

(4-5 order of magnitude short of BEC)