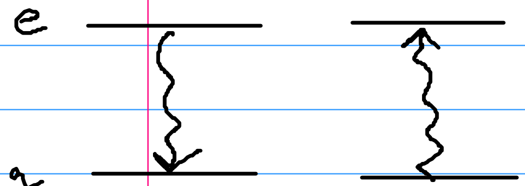
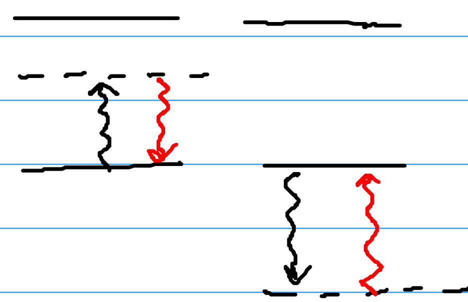
Feynman diagrams

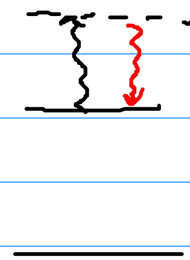
We are Familiar with



Emission Absorption

What about?



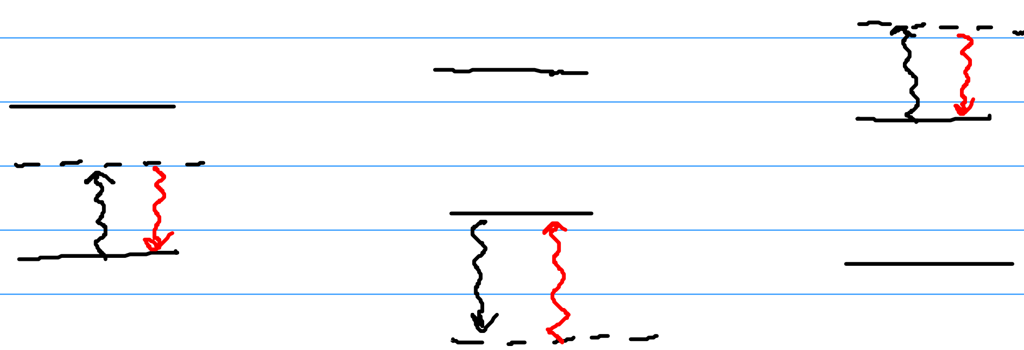


Note: 2nd photon needed
to conserve energy

Virtual states: 

What is their energy?
What is their wavefunction?

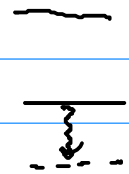
diagrams with time axis:



Correspond to

time

Note:  co-rotating term

 counter-rotating term

"Rules": every photon changes $g \leftrightarrow e$

$|g\rangle \longrightarrow |e\rangle$ by photon absorption OR emission

energy of virtual state reflects photon energy, i.e. $E_g \pm \hbar\omega$

Perturbative calculation of transition amplitudes

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$$H = \underbrace{H_0}_{\text{Atom}} + \underbrace{V}_{\substack{\text{Field} \\ E_\perp}}$$

Interaction picture $|\tilde{\psi}(t)\rangle = e^{iH_0 t/\hbar} |\psi(t)\rangle$

$$\tilde{A}(t) = e^{iH_0 t/\hbar} A \, e^{-iH_0 t/\hbar}$$

$$|\tilde{\psi}(t_f)\rangle = \tilde{U}(t_f, t_i) |\tilde{\psi}(t_i)\rangle$$

Perturbative expansion of $\tilde{U}$

$$\tilde{U}(t_f, t_i) = \mathbb{1} + \sum_{n=1}^{\infty} \underbrace{\tilde{U}^{(n)}(t_f, t_i)}_{\propto V^n}$$

$$i\hbar \frac{d}{dt_f} \underbrace{|\tilde{\psi}(t_f)\rangle}_{\tilde{U}(t_f, t_i)|\tilde{\psi}(t_i)\rangle} = \tilde{V}(t_f)\,|\tilde{\psi}(t_f)\rangle$$

$$\Rightarrow i\hbar \frac{d}{dt_f} \tilde{U}(t_f, t_i) = \tilde{V}\,\tilde{U}(t_f, t_i)$$

Operator equation for $\tilde{U}$ $\Leftrightarrow$ S. eq.

Formal solution

$$\tilde{U}(t_f, t_i) = \mathbb{1} + \frac{1}{i\hbar} \int_{t_i}^{t_f} dt\, \underline{\tilde{V}(t)}\; \tilde{U}(t, t_i)$$

iterative solution $\tilde{U} = \mathbb{1} + \sum \tilde{U}^{(n)}$

0 th order $\mathbb{1}$

1st order $\tilde{U} = \mathbb{1} + \frac{1}{i\hbar} \int dt\, \tilde{V}\; \mathbb{1}$

2nd order $\tilde{U}^{(2)} = \left(\frac{1}{i\hbar}\right)^2 \iint d\tau_1\, d\tau_2\, \tilde{V}\,\tilde{V}$

$$\tilde{U}^{(n)}(t_f, t_i) = \left(\frac{1}{i\hbar}\right)^n \int_{t_f \geq \tau_n \geq \dots \geq \tau_2 \geq \tau_1 \geq t_i} d\tau_n \dots d\tau_2 \, d\tau_1 \; \tilde{V}(\tau_n)\, \tilde{V}(\tau_{n-1}) \quad \tilde{V}(\tau_2)\tilde{V}(\tau_1)$$

Back to Schroedinger equation

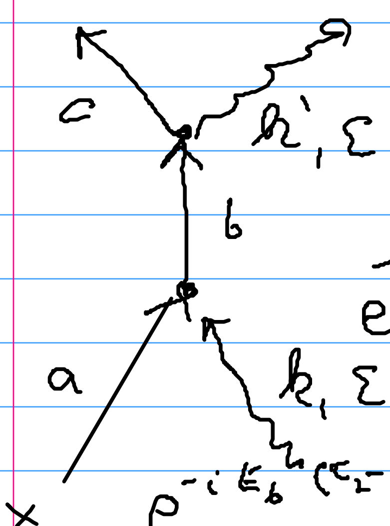
$$\langle \varphi_f | U(t_f, t_i) | \varphi_i \rangle = \delta_{fi} \, e^{-iE_i(t_f - t_i)/\hbar}$$

$$+ \sum_{n=1}^{\infty} \left(\frac{1}{i\hbar}\right)^n \int_{t_f \geq \tau_n \geq t_i} d\tau_n \dots d\tau_1$$

$$\times \sum \langle \varphi_f | V | \varphi_{n-1} \rangle \, e^{-iE_{n-1}(\tau_n - \tau_{n-1})/\hbar}$$

$$\dots \dots \underbrace{\langle \varphi_2 | V | \varphi_1 \rangle}_{\text{Vertex}} \underbrace{e^{-iE_1(\tau_2 - \tau_1)\hbar}}_{\text{Free propagator}} \langle \varphi_1 | V | \varphi_i \rangle e^{-iE_i(\tau_1 - t_i)}$$

e.g.



$$e^{-i\frac{(E_c + \hbar\omega')}{\hbar}(t_f - \tau_2)} \langle c, k' \varepsilon' | V | b, 0 \rangle$$

$$e^{-iE_b(\tau_2 - \tau_1)/\hbar} \langle b, 0 | V | a, k \varepsilon \rangle \, e^{-i(E_a + \hbar\omega)(\tau_1 - t_i)/\hbar}$$

Energy conservation, S & T matrix

$$T = t_f - t_i \to \infty \quad \Rightarrow E_i = E_f$$

e.g. 2nd order perturbation theory

$$\frac{1}{\pi} \frac{\sin (E_f - E_i) T / 2\hbar}{E_f - E_i} \longrightarrow \delta(E_f - E_i)$$

$$S_{fi} = \langle \varphi_f | U(t_f, t_i) | \varphi_i \rangle \quad S \text{ matrix}$$

$$= \delta_{fi} + \sum_{n=1}^{\infty} S_{ki}^{(n)}$$

$$= \delta_{fi} - 2\pi i \, \delta(E_f - E_i) \, \tau_{fi} \quad T \text{ matrix}$$

Transition probabilities

$$P_{fi}(T) = |S_{fi}|^2 = 4\pi^2 (\delta^2(E_f - E_i)) \, |T_{fi}|^2$$

$$+ \int dE_f \longrightarrow \frac{T}{2\pi\hbar}$$

transition rate

2nd order $\quad T_{fi} = \left[V_{fi} + \sum_k \frac{V_{fk} V_{ki}}{E_i - E_k} \right]$

Discussion:

Contribution of intermediate state k E_f

$$\Delta E_k = E_k - E_i$$

$$e^{-i\Delta E_k(\tau_{k+1} - \tau_k)/\hbar} \qquad d\tau \ldots$$

only contribution when $\tau_{k+1} - \tau_k \quad \dfrac{\hbar}{\Delta E_k}$

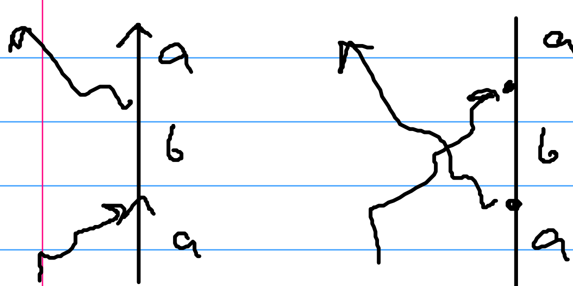
Heisenberg's uncertainty relation

qm system can violate energy conservation for short times, populating off-resonant states

Light scattering

$V = \vec{E}\cdot\vec{d}$ interaction

$\vec{A}\cdot\vec{p}$ $\qquad\qquad\qquad \vec{A}^2$



2nd order in V

1st order

no intermediate states

Thomson scattering $\hbar\omega \gg E_b - E_a$

with $\vec{A}$: $\vec{A}^2$ term dominates

BUT: $\vec{E}\cdot\vec{d}$: same result