Magnetic trapping

Magnetic Trapping

Principle

- Magnetic deflection
  Stern & Gerlach 1925
- Magnetic Focusing
  Friedburg & Paul 1951

1st Suggestions for magnetic trapping
Heer 1963
Vladimirski 1960

1st realization

n	Paul	1978
Na	Phillips	1985
Na	Pritchard	1987
H	Greytak, Kleppner	1987

Earnshaw theorem

Trapping requires $\vec{\nabla}\vec{F} < 0$ or $\Delta\phi > 0$

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Rigid charge distribution

$R + r_i$

↑ com ↑ position of charge q_i

$$\Phi_{tot} = \sum \phi_i = \sum q_i \, \Phi(R + r_i)$$

↑ ext. potential

$$\Delta_R \Phi_{tot} = 0$$

Earnshaw theorem

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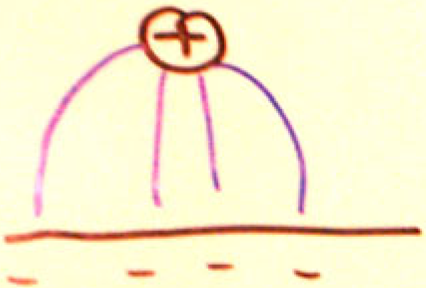
Arbitrary rigid charge distribution (with arbitrary multipole moments) cannot be kept in stable equilibrium at rest in free space in static E fields

Same for magnetic multipole distributions!

Polarization effect may change electric fields [3]

[Sir James Jeans 1925]



Can this effect stabilize a trap?

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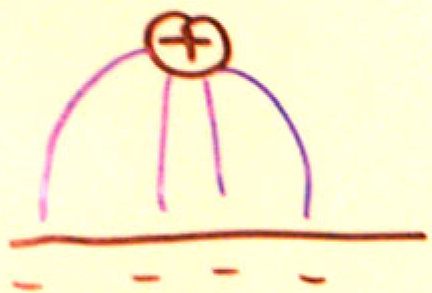
[Sir James Jeans 1925]



Can this effect stabilize a trap?

Loophole: Displacement of the trapped particle or of the field generating particle which are energetically favorable are forbidden due to constraints.

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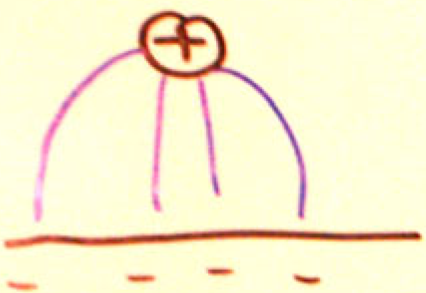

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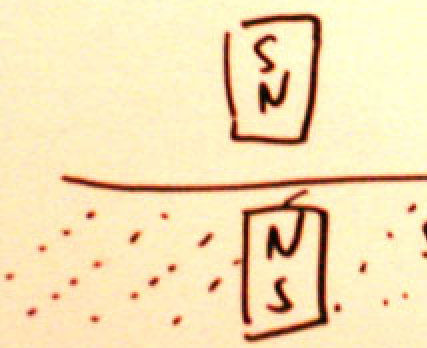
Ex 1:



diamagnetic matter
- expels external field
- internal motion <u>increases</u> total energy

Ex 2: Feedback trap

Ex 3: $\vec{\mu} \cdot \vec{B} = \text{const}$

$\Rightarrow U = (\ )|\vec{B}|$

- classical precession
- plasma physics — dipole moment due to cyclotron motion
- qm. eigenfunction
  $m_F = \text{const.}$

Note:

If $U = \mu_z B_z$ $\Rightarrow \Delta B_z = 0$

no trap

z: lab frame

Wing's theorem (1983)

In a region devoid of charges and currents, quasistatic electric or magnetic fields can have local minima, but not local maxima

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Non-existence proof by contradiction:

$$\vec{E}(r) = \vec{E}(0) + \delta\vec{E}$$

↳ 'Maximum'

$$|E(r)|^2 = \vec{E}^2(0) + 2\vec{E}(0)\,\underline{\delta\vec{E}(r)} + \left(\delta\vec{E}(\vec{r})\right)^2$$

↳ for max: $\vec{E}(0)\,\delta\vec{E}(r) < 0$

↳ assume $\parallel \hat{e}_z$

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↳ For max: $\vec{E}(0)\,\delta\vec{E}(r) < 0$

↳ assume $\parallel \hat{e}_z$

$\Rightarrow \delta E_z(r) < 0$ for small region around 0

$\neq$ to $\Delta \delta E_z = 0$

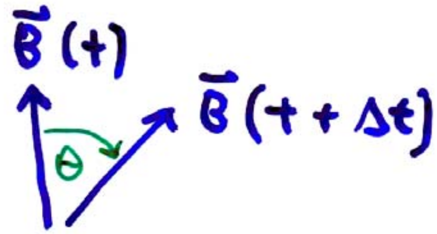
Reminder: $\Delta \psi = 0 \Rightarrow \psi(0) = \overline{\psi(r)}$ — averaged over small sphere around 0

Magnetic trapping
requires $\vec{\mu}$ antiparallel to $\vec{B}$

⇒ Limits to stability

- Spin relaxation
  (involves nuclear magnetic moment)
- Dipolar relaxation
  (involves angular momentum of atomic motion)
- Majorana flops

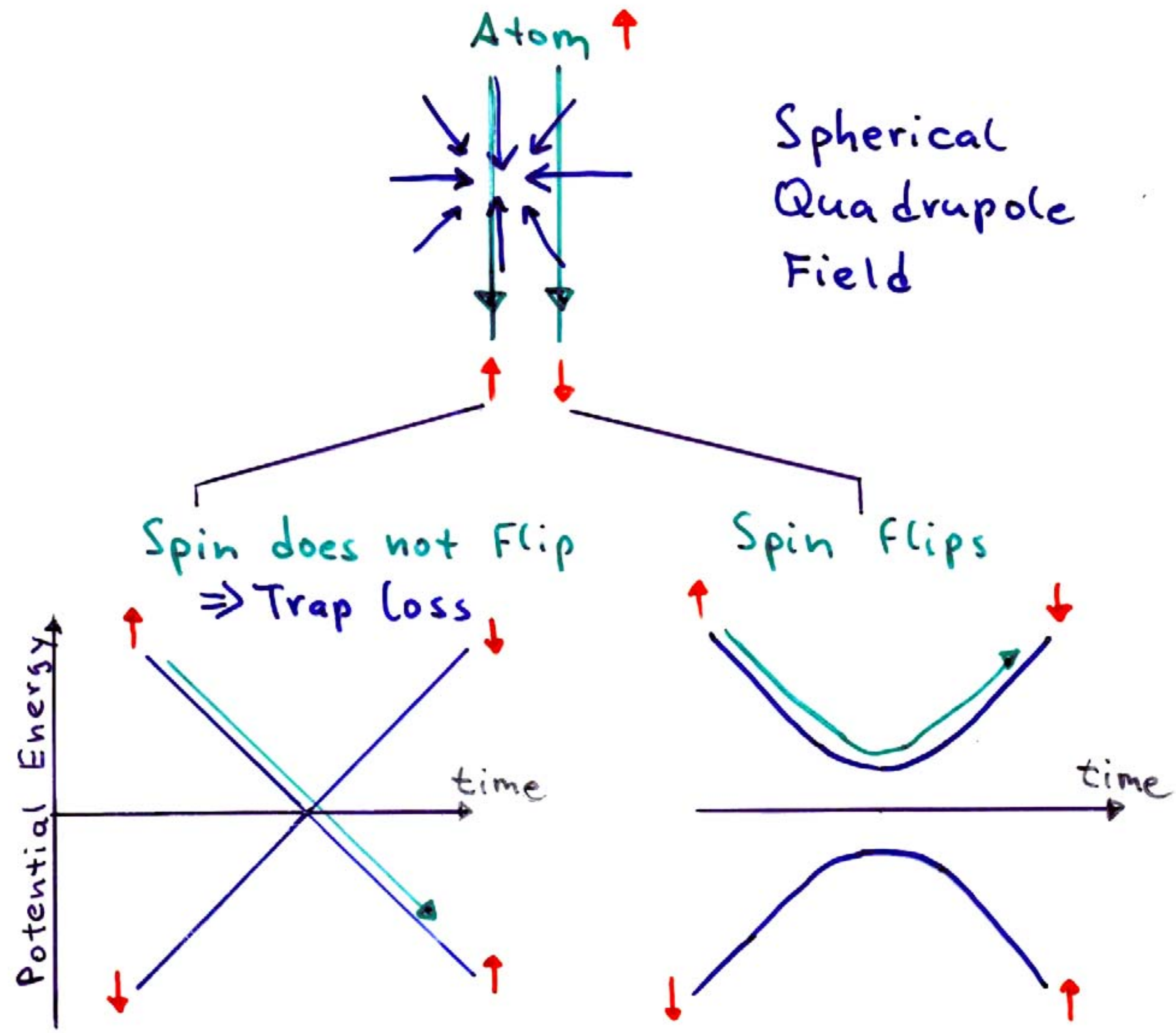
Adiabatic condition



$$\omega_{ROT} = \frac{d\theta}{dt} \ll \frac{U_i - U_j}{\hbar} = g\mu_B B/\hbar$$

$$= \omega_{Larmor}$$

Majorana Flops



Landau-Zener crossing

Levitron movie

Magnetic Trap

$U = \mu |\vec{B}|$

cylindrical symmetry

$r = 0$: $B(z) = B_0 + B' z + B'' z^2/2$

B' → compensates gravity

B'' → > 0

Magnetic Trap

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cylindrical symmetry

$r = 0$: $\quad B(z) = B_0 + B' z + B'' z^2/2$

B' → compensates gravity

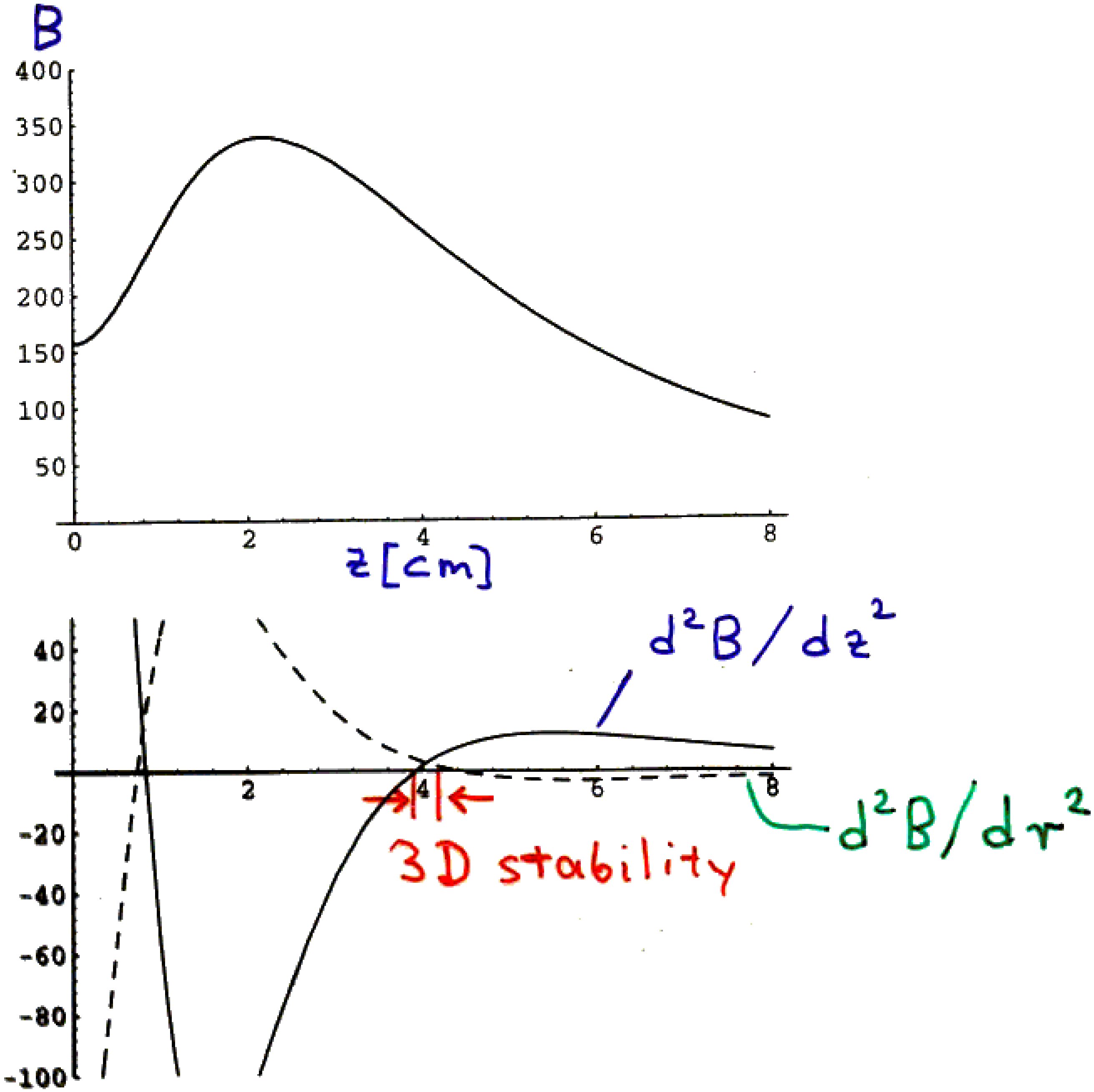
B'' → > 0

$r \neq 0$: $\quad |B| = \sqrt{B_z^2 + B_r^2}$

$= \sqrt{B_z^2 + (B' r/2)^2}$

$\simeq B_z + \frac{1}{8} \frac{B'^2}{B_0} r^2$

$$\frac{d^2 |\vec{B}|}{dr^2} = -\frac{1}{2} B'' + \frac{1}{4} \frac{B'^2}{B_0} > 0$$



B
400
350
300
250
200
150
100
50
0
2
4
6
8
z[cm]
40
20
-20
-40
-60
-80
-100
d^2B/dz^2
d^2B/dr^2
3D stability

Stability of magnetic {trapping / levitation}

- $\vec{\mu}$ has to stay anti-|| to $\vec{B}$
- This happens due to precession as long as

$$\omega_{prec} > \text{rate of change of } \hat{B} \approx \omega_{osc}$$

Stability of magnetic {trapping, levitation}

- $\vec{\mu}$ has to stay anti-|| to $\vec{B}$
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$$\omega_{prec} > \text{rate of change of } \hat{B} \approx \omega_{osc}$$

- $$\omega_{prec} = \frac{\text{max. torque}}{\text{ang. momentum}} = \frac{\mu B}{I\,\omega_{spin}}$$

  I → moment of inertia

- $\omega_{prec} > \omega_{osc}$ is violated for large ω_{spin} or $B \approx 0$

  ⇓

  "Spinflips"
  "Majorana Flops"

Magnetic Traps

Potential U

$$U = -\vec{\mu} \cdot \vec{B} = -\mu B \cos\theta$$

$\cos\theta$ = const (precession)

q.m.: $U = \mu_B B g m_F$

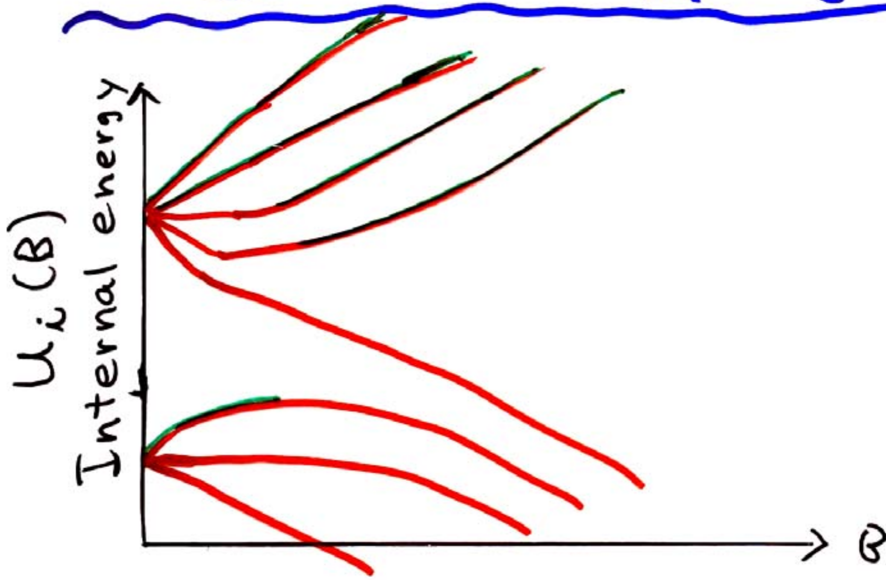
(g: g factor)

B can only have local minima, not maxima!

$\Rightarrow$ choose $\cos\theta < 0$, e.g. -1

$$U = \mu B$$

Magnetic trapping



Stability: $\vec{\nabla} U_i(B(\vec{r})) = 0$

$\Delta U_i(B(\vec{r})) > 0$

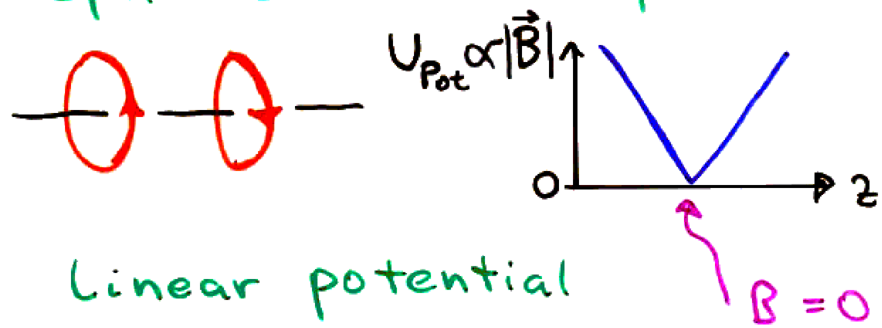
Unless $\frac{d U_i(B)}{dB} = 0$

$\Rightarrow$ Local ~~extremum~~ minimum of $|\vec{B}|$ ← Wing's theorem

→ Only weak-field seeking states

Magnetic Traps

- Spherical Quadrupole



Linear potential

$\Rightarrow$ leaky!

Magnetic Traps

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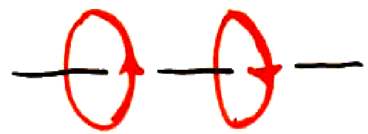

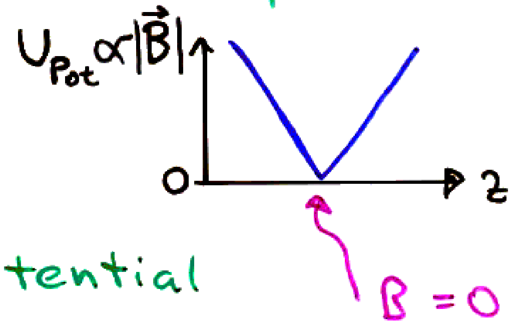

Linear potential

⇒ leaky!

Solutions:

rotating B Field
"TOP" trap
JILA

Optical plug

MIT

TOP trap (JILA '94)

$$\vec{B}_{Stat} = B' \begin{pmatrix} x \\ y \\ -2z \end{pmatrix}$$ Quadrupole Field

$$\vec{B}_{rot} = B_0 \begin{pmatrix} \cos \omega t \\ \sin \omega t \\ 0 \end{pmatrix}$$ rotating bias Field

TOP trap (JILA '94)

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Time-averaged potential

$$U_{TOP} = \frac{\mu}{2} \left(B_r'' r^2 + B_z'' z^2 \right)$$

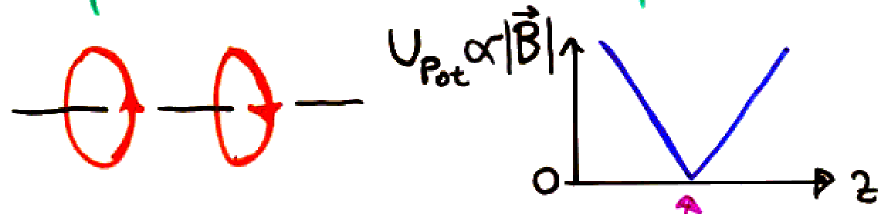
$$B_r'' = \frac{B'^2}{2B_0} \qquad B_z'' = \frac{4B'^2}{B_0}$$

Circle of death $r_D = B_0 / B'$

$$U_{TOP}(r_D) = \mu B_0 / 4$$

Magnetic Traps

- Spherical Quadrupole



Linear potential

B = 0

$\Rightarrow$ leaky!

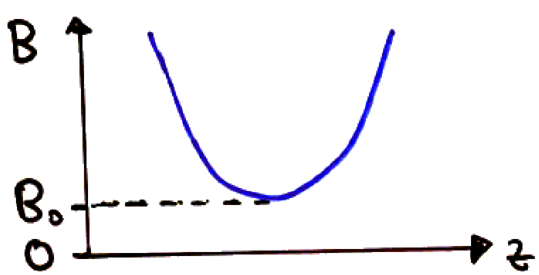
Solutions:

rotating B Field
"TOP" trap
JILA

optical plug

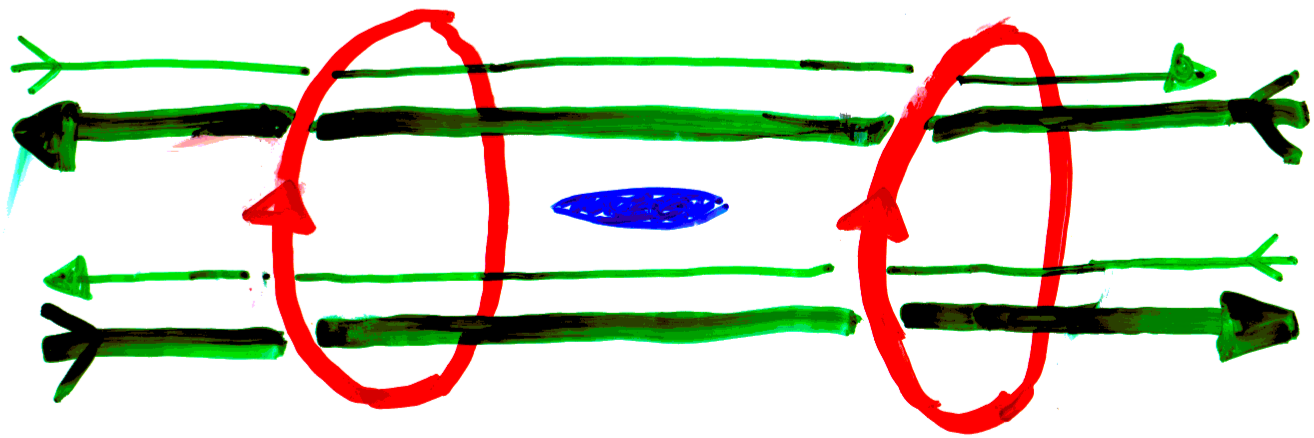
MIT

- Ioffe-Pritchard Trap



harmonic potential

Ioffe-Pritchard Trap



4 "Ioffe" bars

$\Rightarrow B'_{radial}$

2 "Pinch" coils

$\Rightarrow B_0, B''_{axial}$

Ioffe-Pritchard trap

Pinch coil

$$B_z(z) = B_0 + \frac{B''}{2} z^2$$

Ioffe-Pritchard trap

Pinch coil

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Ioffe Bars

2D quadrupole field

$$\begin{pmatrix} B_x \\ B_y \\ B_z \end{pmatrix} = \begin{pmatrix} B' x \\ -B' y \\ 0 \end{pmatrix} \qquad B_\rho$$

$$B = \sqrt{B_z^2 + B_\rho^2} = B_0 + \frac{B''}{2} z^2 + \underbrace{\frac{1}{2} \frac{B'^2}{B_0} (x^2 + y^2)}_{\frac{1}{2} B_\rho^2 / B_0}$$

$B_0 \neq 0$ traps

Ioffe-Pritchard configuration

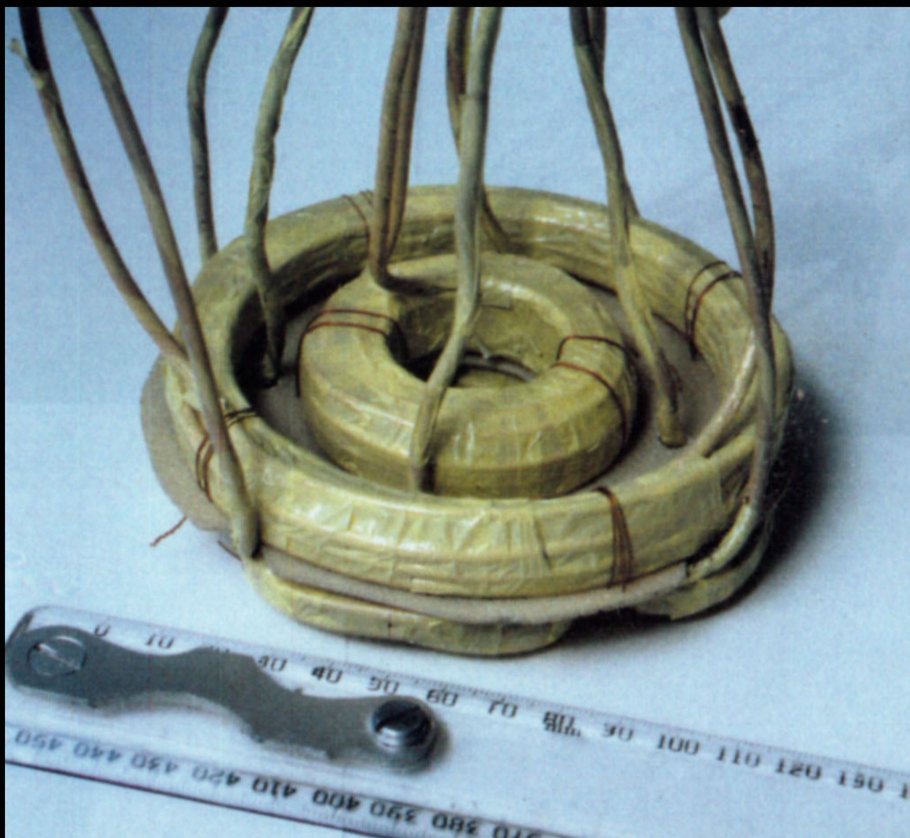
$$\vec{B} = B_0 \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} + B' \begin{pmatrix} x \\ -y \\ 0 \end{pmatrix} + \frac{B''}{2} \begin{pmatrix} -zy \\ -zy \\ z^2 - \frac{1}{2}(x^2+y^2) \end{pmatrix}$$

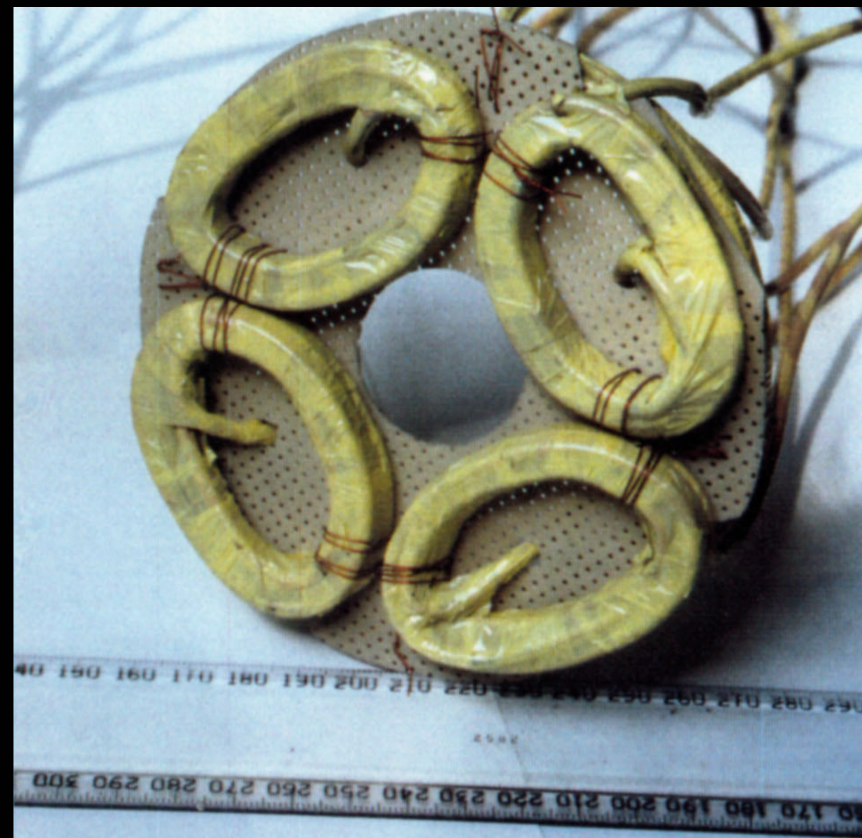
$$B(x,y,z) = \left\{ \left[B_0 + \frac{1}{2} B'' z^2 - \frac{B''}{4}(x^2+y^2) \right]^2 + \left(B' - B'' z/2\right)^2 x^2 + \left(B' + B'' z/2\right)^2 y^2 \right\}^{\frac{1}{2}}$$

$$\approx \frac{1}{2} \left[\frac{B'^2}{B_0} r^2 + B'' z^2 \right]$$

$\uparrow$ large B_0, B' $\qquad$ $\frac{B'^2}{B_0} = B''_{radial}$

WARNING: Green terms limit trapping volume! (important for hot clouds)





Evaporative cooling

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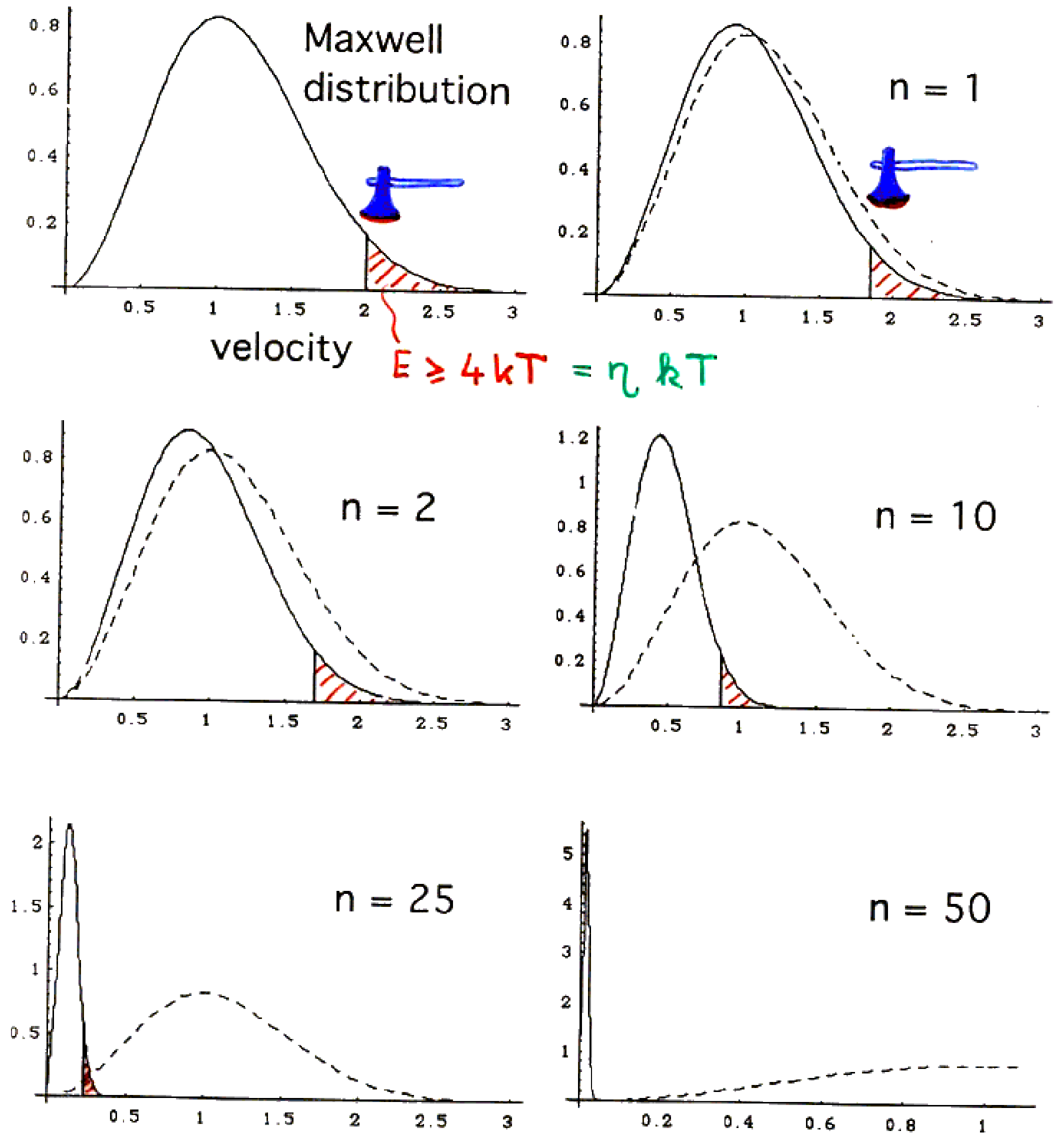

Hess, Phys. Rev. B 34, 3476 (1986).

Proceed with the same factor

$\Rightarrow$ exponential scale

$$\frac{\Delta T}{T} = \alpha \frac{\Delta N}{N} \qquad \alpha = \frac{d(\ln N)}{d(\ln T)} \Rightarrow T = N^{-\alpha}$$

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All other quantities are also power laws of N

Potential $u(r) \sim r^{d/\delta}$ d dimensions

$T \sim r^{d/\delta} \Rightarrow r \sim T^{\delta/d}$

$\mathrm{Vol} \sim T^{\delta} \sim N^{\delta\alpha}$

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$\text{Vol} \sim T^{\delta} \sim N^{\delta\alpha}$

Quantity	Exponent
V	$\delta\alpha$
n	$1-\delta\alpha$
$D \sim n/T^{3/2}$	$1-\alpha(\delta+3/2)$
coll rate $\sim n\sigma v$	$1-\alpha(\delta-1/2)$
T	α

Evaporation controls depth of potential ηkT

ηkT

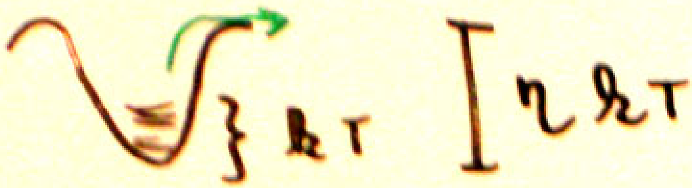
Simple analytical model η = const

Energy of escaping atoms

$$(\eta + \kappa) kT$$

↑ 0 1

Evaporation controls depth of potential ηkT



Simple analytical model $\qquad \eta$ = const

Energy of escaping atoms

$$(\eta + \kappa)\, kT$$

κ: 0 1

total energy $\quad (\delta + \frac{3}{2})\, kT N \qquad \delta = \frac{3}{2}$ HO

δ ↑ pot $\quad \frac{3}{2}$ ↑ kinetic

Energy change during dt

$$\left(\delta + \frac{3}{2}\right) kT N + dN (\eta + \kappa) kT$$

$$= \left(\delta + \frac{3}{2}\right) k (T + dT) (N + dN)$$

Energy change during dt

$$\left(\delta + \tfrac{3}{2}\right) kTN + dN(\eta + \kappa) kT$$

$$= \left(\delta + \tfrac{3}{2}\right) k (T + dT) \; (N + dN)$$

neglect

$$\Rightarrow \frac{dT}{T} = \frac{dN}{N} \underbrace{\left(\frac{\eta + \kappa}{\delta + 3/2} - 1 \right)}_{\alpha}$$

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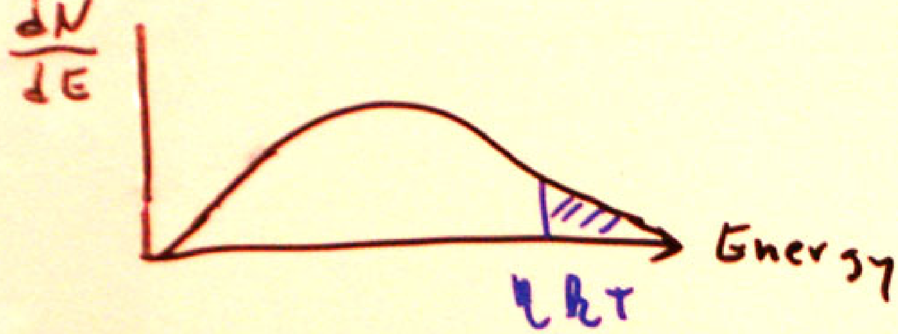
α characterizes how much more than the average energy $\left(\delta + \frac{3}{2}\right) kT$ is removed by escaping atom

How efficient can evaporative cooling be?

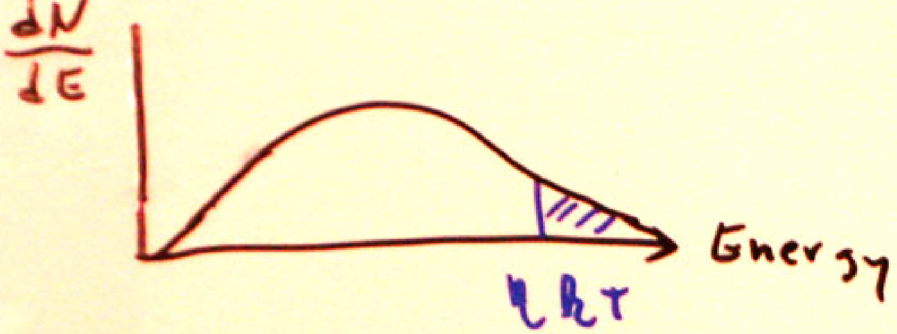
Speed of evaporation and loss processes



detailed balance (large η):

rate of evaporation = rate of collisions of atoms in the wing

Speed of evaporation and loss processes



detailed balance (large η):

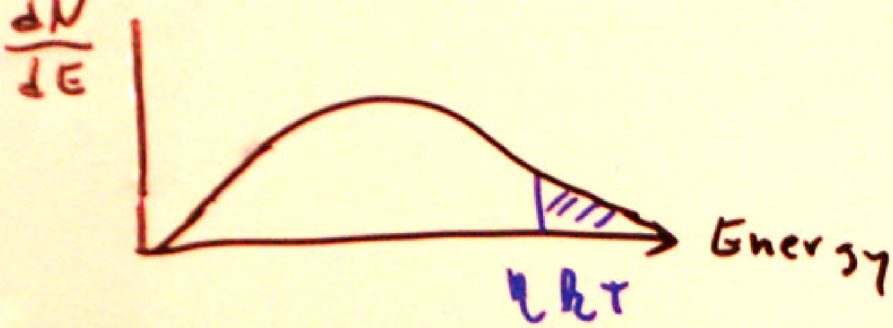
rate of evaporation = rate of collisions of atoms in the wing

Fraction of atoms with $E > \eta kT$

$$2 e^{-\eta} \sqrt{\eta/\pi}$$

Velocity: $v = \sqrt{2\eta kT/m} = \sqrt{\pi\eta}\,\frac{\bar{v}}{2}$ — average thermal velocity

Speed of evaporation and loss processes



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$$\dot{N} = -n_0 \sigma v \left[N \cdot 2 \cdot e^{-\eta} \sqrt{\eta/\pi} \right] = -N/\tau_{ev}$$

Compare to $\frac{1}{\tau_{el}} = n_0 \sigma \bar{v} \sqrt{2}$

$$\lambda = \frac{\tau_{ev}}{\tau_{el}} = \frac{\sqrt{2}\, e^{\eta}}{\eta}$$

$$\frac{\dot{N}}{N} = -\frac{1}{\lambda \tau_{el}}$$

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$$\frac{\dot{N}}{N} = -\frac{1}{\lambda \tau_{el}}$$

Two parameters for evaporation

$\alpha(\eta)$ → efficiency of evaporation

$\lambda(\eta)$ → # of collisions necessary to result in evap.

better | worse

for large η

Runaway evaporation

$$\frac{d(n\sigma v)}{n\sigma v} = \left(1 - \alpha\left(\delta - \tfrac{1}{2}\right)\right) d(\ln N)$$

$$= -\frac{\left(1 - \alpha\left(\delta - \tfrac{1}{2}\right)\right)}{\lambda}\frac{dt}{\tau_{el}} - \frac{dt}{\tau_{loss}}$$

← Simple loss term indep. of v, n

Runaway evaporation

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Simple loss term indep. of v, n

$$= \frac{1}{\tau_{el}}\left(\frac{\alpha\left(\delta - \tfrac{1}{2}\right) - 1}{\lambda} - \frac{1}{R}\right)$$

$$R = \frac{\tau_{loss}}{\tau_{el}}$$

'ratio of good to bad collisions'

Runaway evaporation

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Simple loss term indep. of v, n

$$= \frac{1}{\tau_{el}}\left(\frac{\alpha\left(\delta-\tfrac{1}{2}\right)-1}{\lambda} - \frac{1}{R}\right)$$

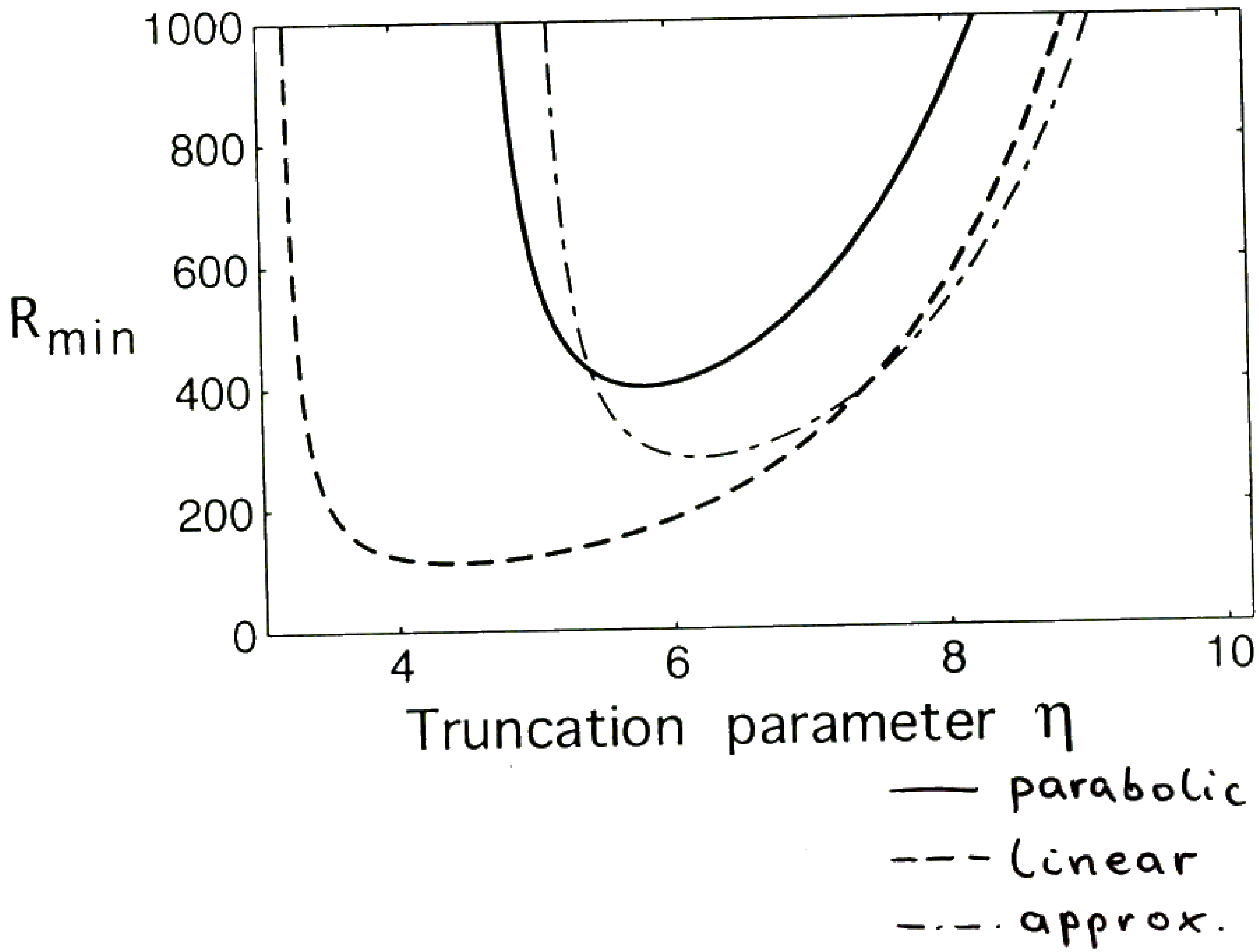
$$R = \frac{\tau_{loss}}{\tau_{el}}$$

'ratio of good to bad collisions'

$R > R_{min} = \dfrac{\lambda}{\alpha\left(\delta-\tfrac{1}{2}\right)-1}$ evap. speeds up

└ typically a few 100

Runaway evaporation

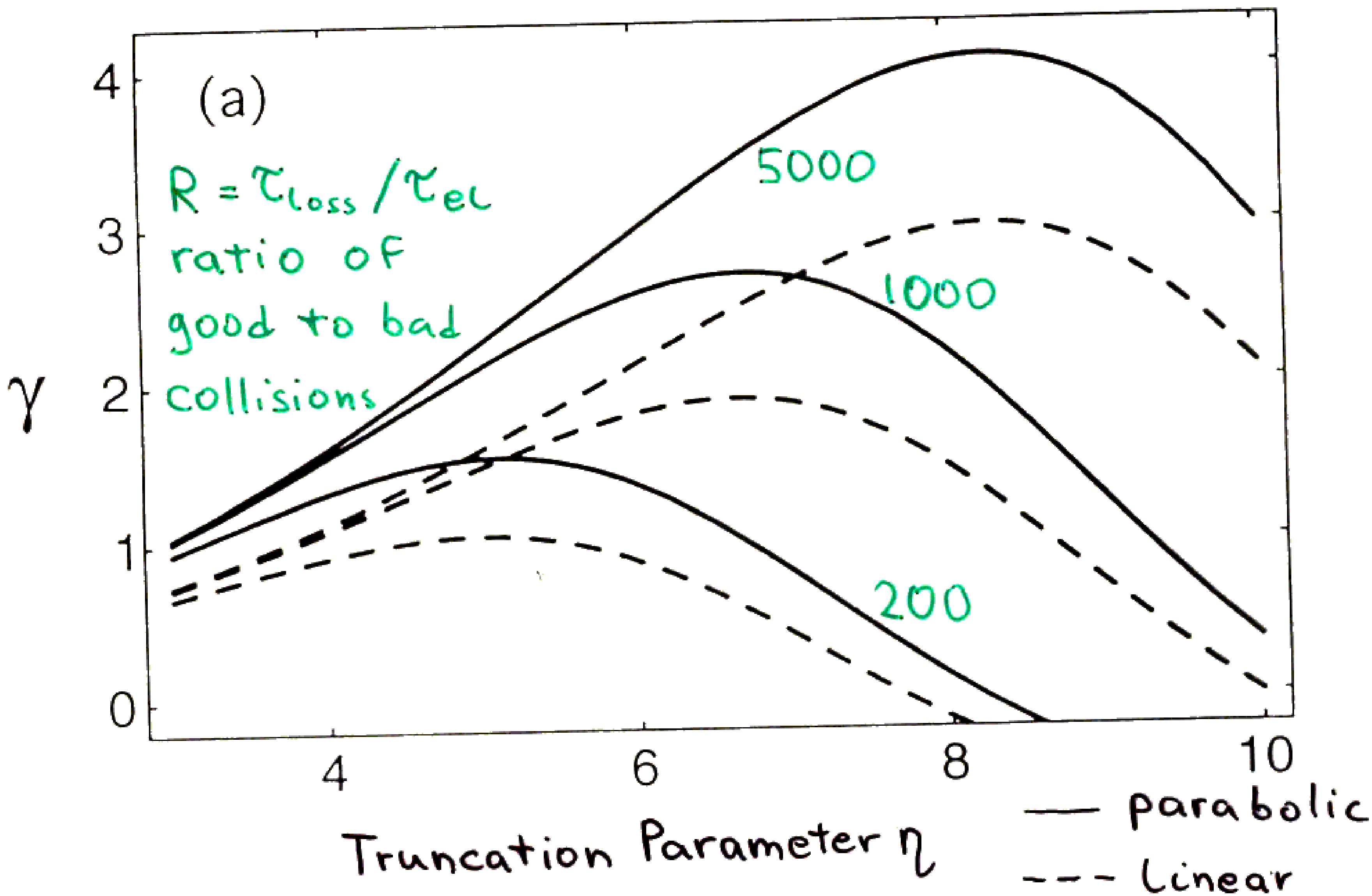


Increase of phase-space density

$$\gamma = -\frac{d(\ln D)}{d(\ln N)} = \frac{\alpha(\delta + \frac{3}{2})}{1 + \lambda/R} - 1$$

Efficiency of evaporative cooling

$$\gamma = -\frac{d(\ln D)}{d(\ln N)} \xrightarrow[R \to \infty]{} \eta$$

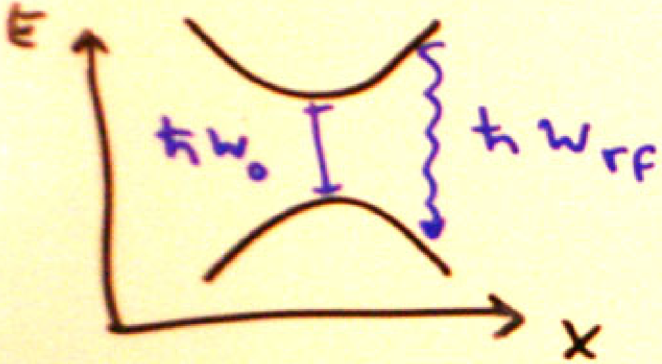


Scenario

$\eta = 6$	$\eta = 5$
0.7% loss per τ_{ee}	
D ↑ 10^6 after 600 collisions	D ↑ 10^6 after 300 collisions
N ↓ 100	N ↓ 500
$\gamma = 3$	$\gamma = 2.2$

RF induced evaporation

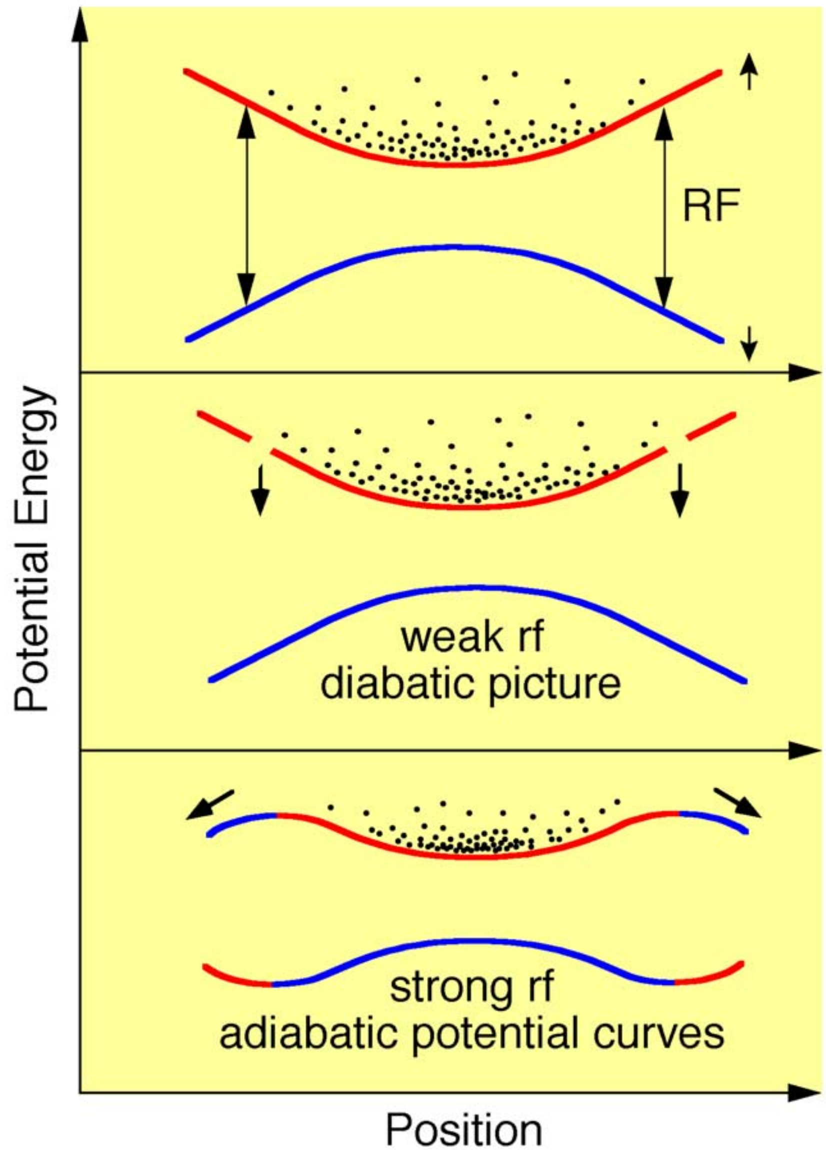
Method of choice in magnetic trap



Atoms with

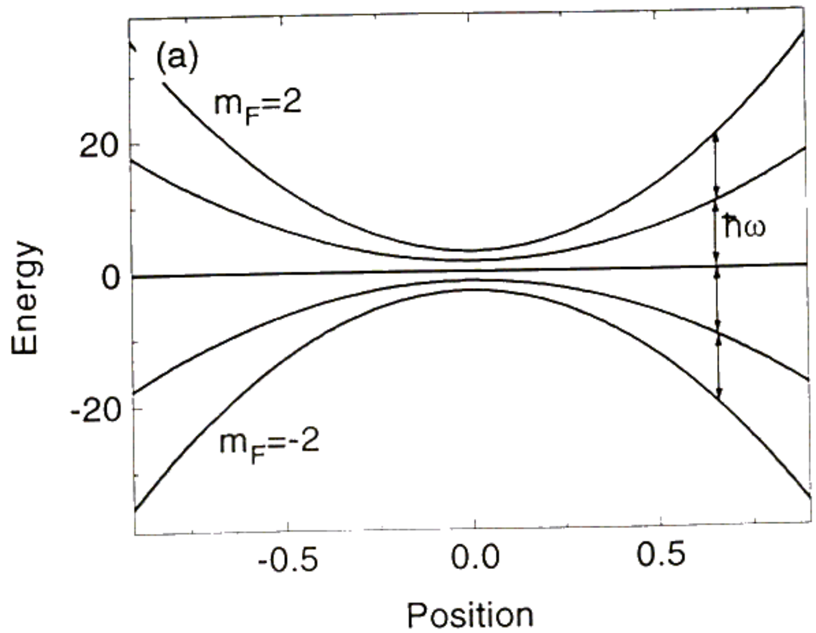
$E > |m_F|\hbar(\omega_{RF} - \omega_0)$ evaporate

Rf induced evaporative cooling

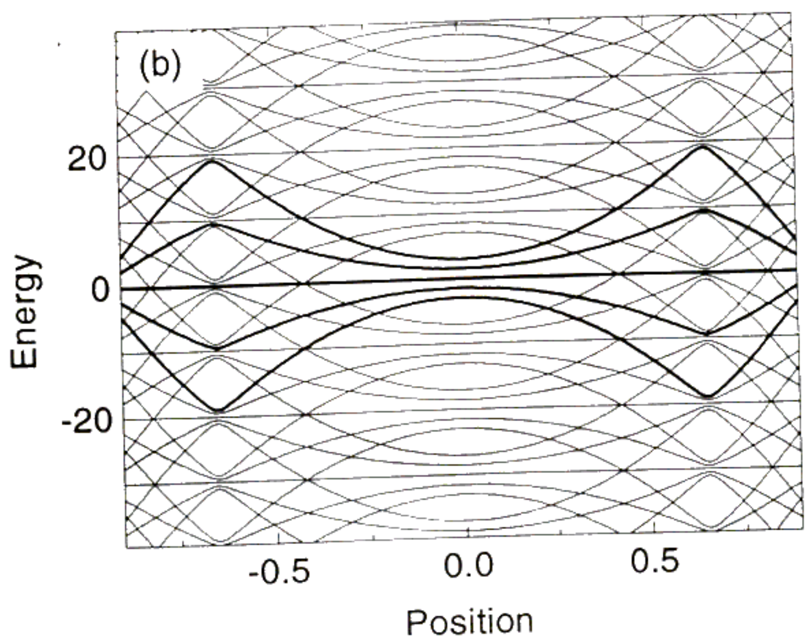


Atoms with energy >$h(\nu_{rf}-\nu_0)/2$ evaporate!

suggestion: Pritchard et al. (1989)
exp.: Ketterle et al. (OSA, 1993)



(a)
$m_F=2$
$m_F=-2$
$\hbar\omega$
Energy
20
0
-20
-0.5
0.0
0.5
Position



(b)
Energy
20
0
-20
-0.5
0.0
0.5
Position

Cooling limit for evaporative cooling?

No fundamental limit
Practical limit: Depends on residual heating process
i.e. secondary collisions, density dependent loss term