LIGHT FORCES

API pp 370-378

$$H_{I1} = -\vec{d}\,\vec{E}_{\perp}$$

$\vec{d}$: atomic dipole operator

$\vec{E}_{\perp}$: operator of the quantized em. field or $\hat{a} + \hat{a}^{\dagger}$

em. field coherent state $|\alpha\rangle$

exercise 17 in API

exact unitary transformation

$$|\alpha\rangle \longrightarrow |0\rangle \quad \text{vacuum state}$$

$$\vec{E}_{\perp} \longrightarrow \vec{\mathcal{E}}_0 \cos \omega_L t + \vec{E}_{\perp}$$

vacuum fluctuations of the em field

Atomic dipole : OBE

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Bloch vector

$$\langle J_x \rangle = u = \frac{1}{2}\left(\hat{\sigma}_{ab} + \hat{\sigma}_{ba}\right)$$
$$\langle J_y \rangle = v = \frac{1}{2i}\left(\hat{\sigma}_{ab} - \hat{\sigma}_{ba}\right)$$
$$\langle J_z \rangle = w = \frac{1}{2}\left(\hat{\sigma}_{bb} - \hat{\sigma}_{aa}\right)$$

$\hat{\sigma}$: atomic density matrix in RWA and rotating frame

$\hookrightarrow$ Fictious spin $\frac{1}{2}$

[eq. A20]

$$\boxed{\begin{aligned} \dot{u} &= \delta_L v - \frac{\Gamma}{2} u \\ \dot{v} &= -\delta_L u - \Omega_1 w - \frac{\Gamma}{2} v \\ \dot{w} &= \Omega_1 v - \Gamma w - \frac{\Gamma}{2} \end{aligned}}$$

$\delta_L = \omega_L - \omega_0$ Laser detuning

Ω_1 Rabi Freq.

$\hbar \Omega_1 = -\vec{d}_{ab} \vec{\mathcal{E}}_0$

$\vec{\mathcal{E}} = \vec{\mathcal{E}}_0 \cos \omega_L t$

Lecture 9

$$\dot{r}_x = \delta r_y - \frac{\Gamma}{2} r_x$$
$$\dot{r}_y = -\delta r_x - g r_z - \frac{\Gamma}{2} r_y$$
$$\dot{r}_z = g r_y - \Gamma (r_z \mp 1)$$

Translation

$\delta \rightarrow \delta_L$

$g \rightarrow \Omega_1$

$r_{x,y,z} \rightarrow 2u, 2v, 2w$

Mechanical Forces $\vec{d} \, \vec{\nabla} E$

$$\vec{d} = \vec{d_{ab}} \left(|b\rangle\langle a| + |a\rangle\langle b| \right)$$

A.21

$$\langle d \rangle = \mathrm{Tr}(\sigma d) = \vec{d_{ab}} \left(\sigma_{ab} + \sigma_{ba} \right)$$

$$= \vec{d_{ab}} \left(\hat{\sigma}_{ab} e^{i\omega_L t} + \hat{\sigma}_{ba} e^{-i\omega_L t} \right)$$

$$= 2 \vec{d_{ab}} \left(u \cos\omega_L t - v \sin\omega_L t \right)$$

in phase   in quadrature

with incident field

Absorbed energy power

$$\frac{dW}{dt} = q \, \mathcal{E}_0 \cos\omega_L t \, \frac{dr}{dt}$$

$\langle \dot{d} \rangle$

$$\left\langle \frac{dW}{dt} \right\rangle = \hbar \Omega_1 \omega_L v \qquad \div \hbar \omega_L$$

averaged over one cycle

$$\rightarrow \left\langle \frac{dN}{dt} \right\rangle = \Omega_1 v$$

absorbed photons

Radiative Force

Center of wavepacket $\vec{r}_G$

$$\vec{F} = \dot{\vec{p}} = -\frac{\partial H}{\partial R} = \sum_{j=x,y,z} d_j \nabla_R \left[E_{ej}(\cancel{\vec{R}}, t) + \cancel{E_{\perp j}(\vec{R})} \right]$$

Heisenberg eq of motion

Vacuum field does not contribute

Assumption that atoms can be well localized (Not valid when cooling to the recoil limit)

$$M \ddot{\vec{r}}_G = \sum_j \langle d_j \rangle \nabla E_{ej}(\vec{r}_G, t)$$

Use u_{st}, v_{st}

Damping time for external motion $\frac{\hbar}{E_{rec}}$

$\gg$ internal motion Γ^{-1}

Na $\times 400$

He triplet comparable

Forces for $v=0$ at $r=0$

$$\vec{E}_e(\vec{r},t) = \hat{e}\;\; \mathcal{E}_0 \cos\left(\omega_L t + \phi(r)\right)$$

Pol. independent of $\vec{r}$

$$\nabla E_{e_j} = e_j\left[\cos\omega_L t\, \nabla\mathcal{E}_0 - \sin\omega_L t\; \mathcal{E}_0\, \nabla\phi\right]$$

$$\langle d_j \rangle = 2(d_{ab})_j\left[u_{ST}\cos\omega_L t - v_{ST}\sin\omega_L t\right]$$

$$\underset{\text{cycle average}}{\vec{F}} = \underbrace{(\hat{e}\, \vec{d}_{ab})\left[u_{st}\nabla\mathcal{E}_0\right.}_{F_{react}} \left. + v_{st}\,\mathcal{E}_0\nabla\phi\right]$$

F_{diss}

$$\Omega_1 = -\vec{d}_{ab}\,\hat{e}\;\mathcal{E}_0/\hbar \qquad \text{Rabi Freq.}$$

$$\vec{\alpha} = \frac{\nabla\Omega_1}{\Omega_1} \qquad\qquad \vec{\beta} = \nabla\phi$$

$$\vec{F}_{react} = -\hbar\,\Omega_1\, u_{st}\,\vec{\alpha}$$

$$\vec{F}_{diss} = -\hbar\,\Omega_1\, v_{st}\,\vec{\beta}$$

Radiation pressure force

Plane travelling wave $\quad \vec{\alpha} = 0$

$$\vec{E}_L = \varepsilon\, \mathcal{E}_0 \cos(\omega_L t - \vec{k}_L \vec{r})$$

$$\vec{\beta} = -\vec{k}_L$$

$$\vec{F}_{diss} = \Omega_1 v_{st}\, \hbar k_L = \left\langle \frac{dN}{dt} \right\rangle_{st} \hbar k_L$$

\# of absorbed atoms

$$= \Gamma\, \sigma_{bb}^{st}$$

$$\vec{F}_{diss} = \hbar \vec{k}_L \frac{\Gamma}{2} \frac{\Omega_1^2/2}{(\omega_L - \omega)^2 + (\Gamma^2/4) + (\Omega_1^2/2)}$$

$$\vec{F}_{diss,max} = \hbar \vec{k}_L \cdot \frac{\Gamma}{2}$$

Reactive Forces

simplest ex. standing wave

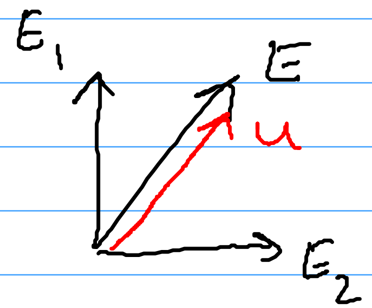
$$\vec{E}_L(\vec{r},t) = \hat{e}_x\, \mathcal{E}_0 \cos k_L z \cos \omega_L t$$

depends only on u $\Rightarrow$ no exchange of energy

BUT: redistribution of energy and mom

SW as superposition of two TWs

pick a point where E_1 and E_2 are 90° phase shifted



if u is within 0 ... 180° with E_1

180 ... 360°

absorb
deliver

E_1 loses energy
E_2 gains energy

$$\vec{F}_{react} = -\frac{\hbar(\omega_L-\omega_0)}{4}\frac{\nabla(\Omega_1^2)}{(\omega_L-\omega_0)^2+\frac{\Gamma^2}{4}+\frac{\Omega_1^2}{2}}$$

Dispersive

red-detuning attractive
blue repulsive

$$\vec{F}_{react} = -\nabla U$$

$$U = \frac{\hbar(\omega_L-\omega_0)}{2}\ln\left[1+\frac{\Omega_1^2/2}{(\omega_L-\omega_0)^2+\Gamma^2/4}\right]$$

dipole potential $\Rightarrow$ dipole traps max Force?

For given Ω_1^2, optimum $|\delta_L| \sim |\Omega_1|$

$$|F_{react}| \sim \frac{\hbar|\nabla\Omega_1^2|}{\Omega_1} \sim \hbar|\nabla\Omega_1| \sim \hbar k_L \Omega_1$$

momentum exchanges of $\hbar k_L$ at rate Ω_1