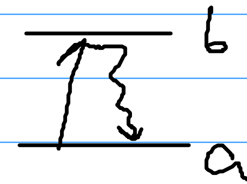
Resonant scattering $\qquad$ API pp 93-96

$$T_{fi} = \langle \Psi_f | V \frac{1}{E_i - H_0} V | \Psi_i \rangle$$

2nd order

$$\frac{1}{E_i - H_0} \to \sum_k |\Psi_k\rangle \frac{1}{E_i - E_k} \langle \Psi_k|$$

Resonant scattering



$\hbar\omega \simeq E_b - E_a$

$$T = \langle a, k' \varepsilon' | H_{I1} \frac{1}{E_a + \hbar\omega - \cancel{H_0}\, H} H_{I1} | a, k \varepsilon \rangle$$

H_{I1}: $A \cdot p$, $d \cdot E$

$$= \frac{\langle a, k'\varepsilon' | H_{I1} | b, 0\rangle \langle b, 0 | H_{I1} | a, k\varepsilon\rangle}{\hbar\omega + E_a - E_b + i\hbar(\Gamma/2)}$$

$$\frac{1}{\hbar(\omega - \omega_0 + i(\Gamma/2))} = \frac{1}{\hbar} \sum_{n=0}^{\infty} (-1)^n \frac{(i\Gamma/2)^n}{(\omega - \omega_0)^{n+1}}$$

Γ 2nd order in H_{I1}

- infinite order in H_{I1}
- non-perturbative

API 167-174
180-189

Evolution operator and resolvent

$$U(t,t') = U_0(t,t') + \frac{1}{i\hbar}\int_{t'}^{t} dt_1\, U_0(t,t_1)\, V\, U(t_1,t')$$

has iterative solution

$\Rightarrow$ perturbative expansion

If integral were a convolution, the F.T. could turn it into a product

But: integral is NOT $\int_{-\infty}^{\infty}$

Define $K(t,t') = U(t,t')\,\Theta(t-t')$
$K_0 \qquad\qquad U_0$

$$K(t,t') = K_0(t,t') + \frac{1}{i\hbar}\int_{-\infty}^{\infty} dt\, K_0 V K$$

convolution

(retarded) Green's Function

$G(E)$ F.T. of K

$$= \frac{1}{i\hbar}\int_{-\infty}^{\infty} d\tau\, e^{iE\tau/\hbar}\, K(\tau)$$

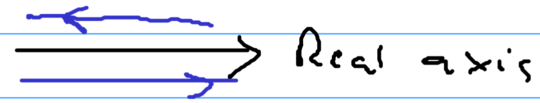
$K(\tau) = e^{-iH\tau/\hbar}\,\Theta(\tau)$

$$= \frac{1}{E-H+i\eta}$$

$$G(E) = G_0(E) + G_0(E)\, V\, G(E)$$

$$G(z) := \frac{1}{z - H} \qquad \text{resolvent}$$

$$U(\tau) = \frac{1}{2\pi i} \oint dz\, e^{iz\tau/\hbar}\, G(z)$$



iterate algebraic equation

$$G \to G_0 + G_0 V G_0 + G_0 V G_0 V G_0 + \dots$$

"geometric series"

$$\frac{1}{z - H_0}$$

$$\left.\begin{matrix} \langle \varphi_k | & | \varphi_l \rangle \\ E_k & E_l \end{matrix}\right\} \text{ of } H_0$$

$$G_{kl} = \frac{1}{z - E_k}\,\delta_{kl} + \frac{1}{z - E_k}\, V_{kl}\, \frac{1}{z - E_l} + \dots$$

Now: $|\varphi_b\rangle$ discrete state of H_0 with energy E_b

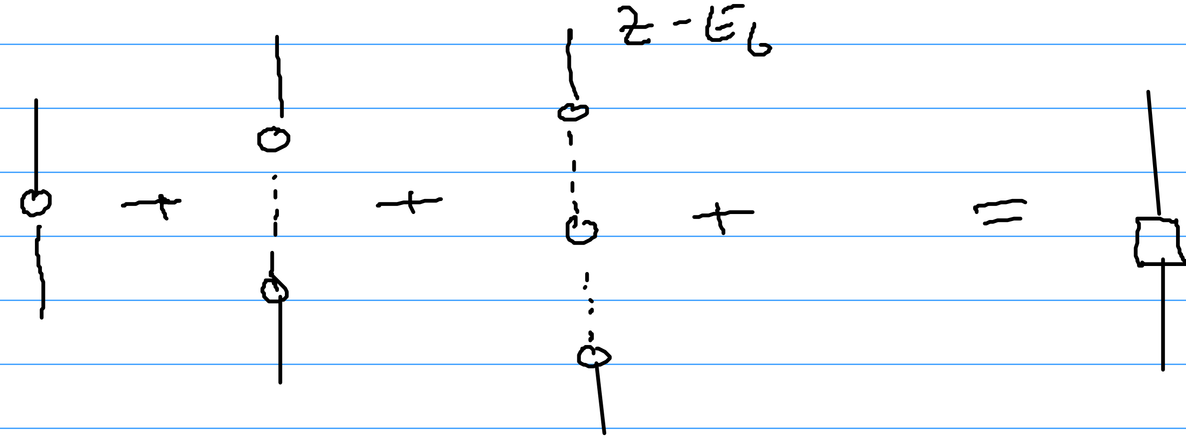
$\Rightarrow$ expansion involves denominators

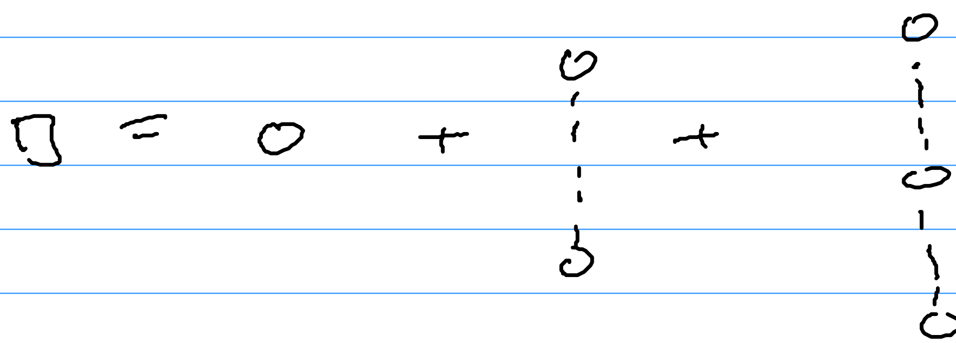
$$\frac{1}{\underset{\uparrow E}{z} - E_b}$$

We are interested in $z \simeq E_b \Rightarrow$ divergence diagrammatic representation for G_{66}

$\circ$ V $\qquad$ $|$ $\frac{1}{z-E_6}$ $\qquad$ $\vdots$ $\frac{1}{z-E_i}$ $\quad i \neq 6$

all terms which contain $\frac{1}{z-E_6}$ twice

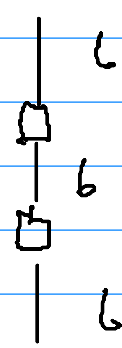




$$=: R_6(z) = V_{66} + \sum_{i\neq 6} V_{6i} \frac{1}{z-E_i} V_{i6} + \dots$$

Next: $\left(\frac{1}{z-E_6}\right)^n$

$n = 3$



Contribution to $G_6(z)$ is $\frac{1}{(z-E_6)^3}\left(R_6(z)\right)^2$

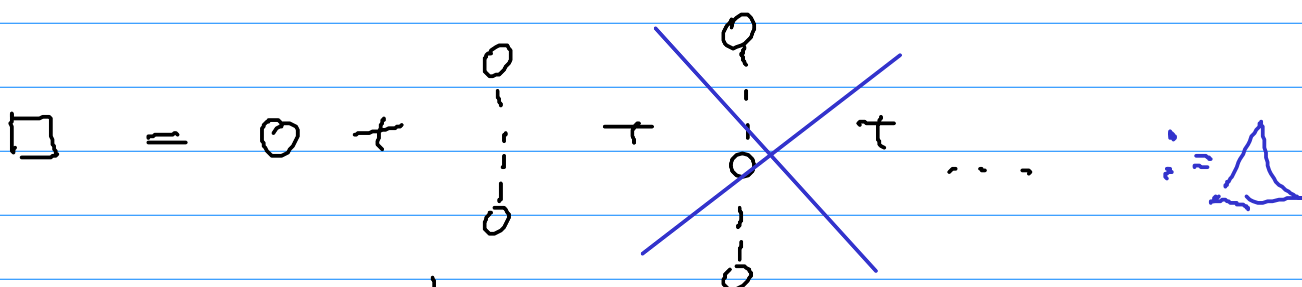
$$\Rightarrow \quad G_6(z) = \frac{1}{z-E_6} \sum_{n=0}^{\infty} \left[\frac{R_6(z)}{z-E_6}\right]^n$$

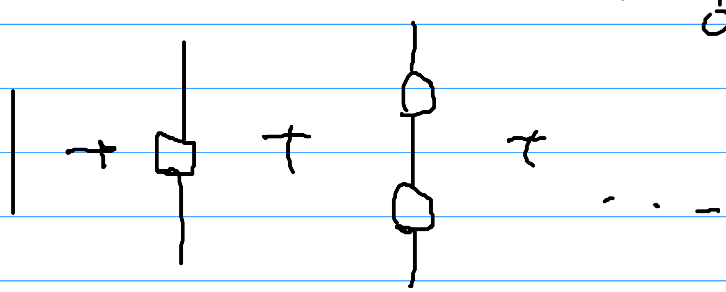
$$= \frac{1}{z-E_6 - \underline{R_6(z)}}$$

exact

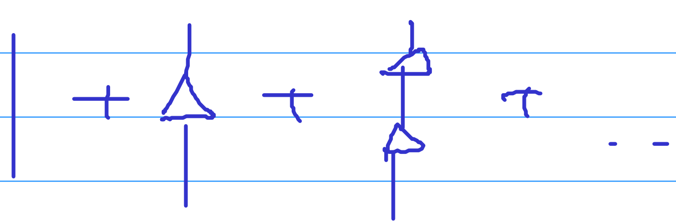
Importance of this result:

- Resonant structure of $G_6(z)$ is accounted for
- $R_6(z)$ has no divergences near $z \approx E_6$
- Simple approximations to $R_6(z)$ are possible and correspond to an infinite number of terms in a perturbative expansion.





approximated by



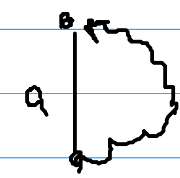
Apply to excited atomic state

$$|\Psi_b\rangle = |b;0\rangle$$

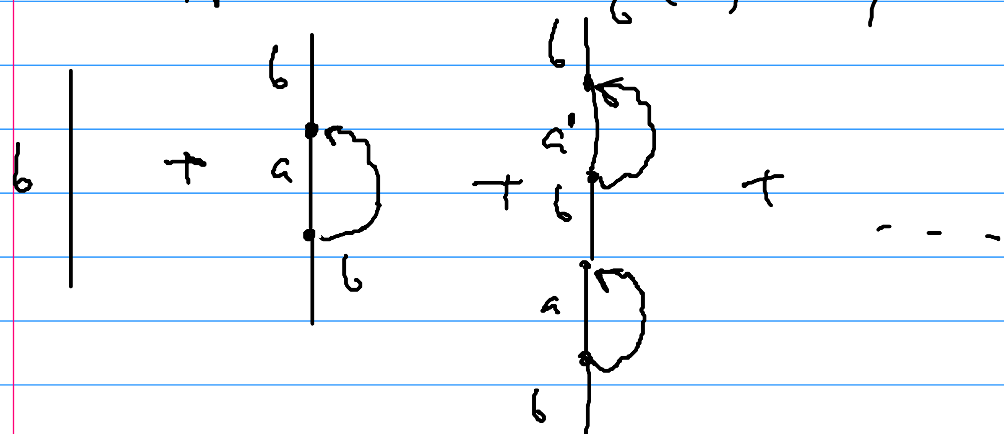
Calculate $G_b(z) \xrightarrow[\text{integration}]{\text{Contour}} U_b(\tau) = \langle b,0|U(\tau)|b,0\rangle$

$$G_b(z) = \frac{1}{z - E_b - R_b(z)}$$

2nd order approx $\hat{R}_b(z) =$

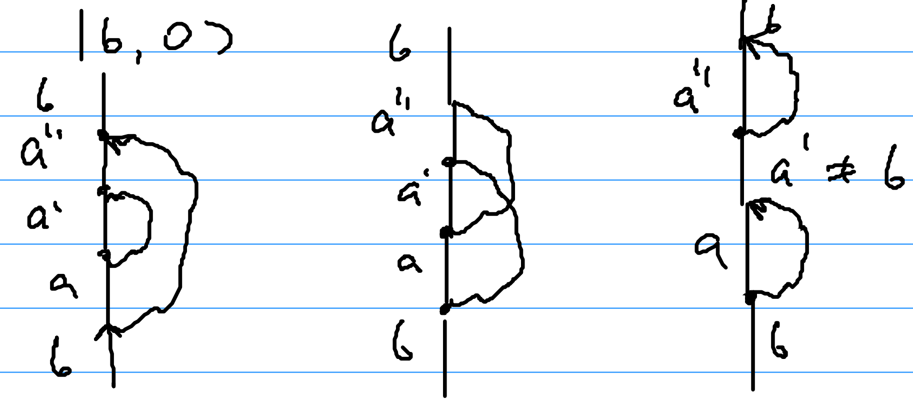


We approximate $G_b(z)$ by



We neglect ALL processes where we have SEVERAL intermediate states between two states $|b,0\rangle$

e.g.



$$\hat{R}_b(\cancel{E}) = \sum_{a} \sum_{k\varepsilon} \frac{\langle a, k\varepsilon | H_{I} | b, 0 \rangle}{E - E_a - \hbar\omega}$$

$$= \hbar \Delta_b(E_b) - i\hbar \Gamma_b(E_b)/2$$

≙ Markov approximation

$$G_b(E) = \frac{1}{E - E_b - \hbar \Delta_b + i\left(\frac{\hbar}{2}\right)\Gamma_b}$$

$$U_b(\tau) = e^{-i(E_b + \hbar\Delta_b)\tau/\hbar} \, e^{-\Gamma_b |\tau|/2}$$

radiative shift — exponential decay

perturbative expansion would not give exponential, but polynominal decay.